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Preparation 10

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Patterns and Algebra

Module 3

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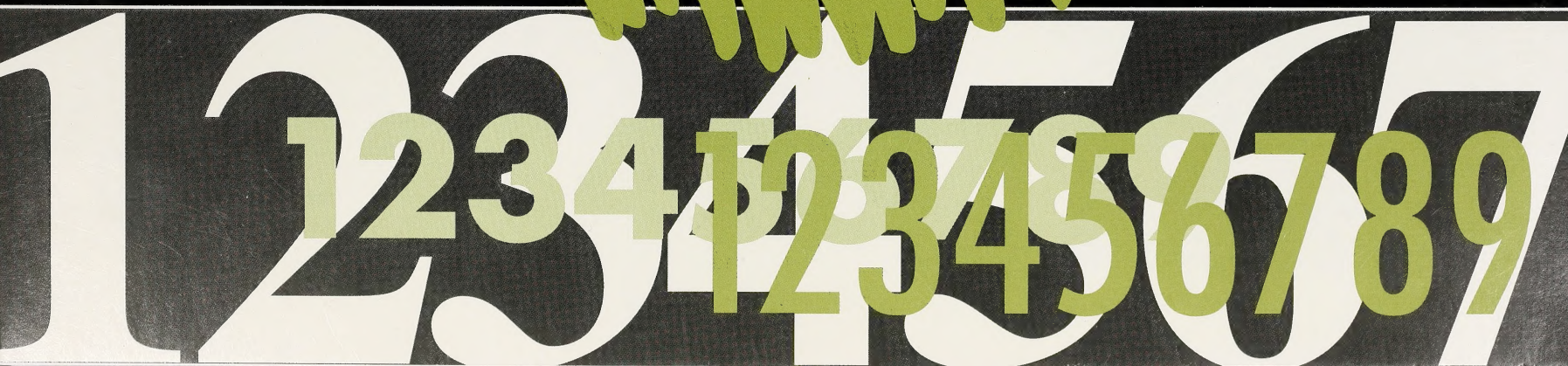

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MATHEMATICS
Preparation 10

Module 3
Patterns and Algebra

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Mathematics Preparation 10
Module 3: Patterns and Algebra
Student Module Booklet
Learning Technologies Branch
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This document is intended for	
Students	✓
Teachers	✓
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Other	

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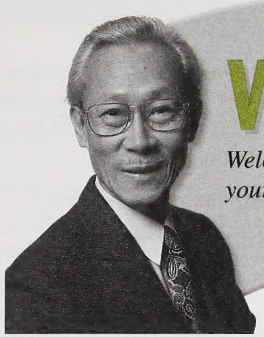

Welcome

Welcome to Module 3. We hope you enjoy your study of "Patterns and Algebra."

Course Overview

Mathematics Preparation 10 consists of five modules and a final test. Each module is worth 1 credit. Discuss with your teacher how many modules you need to complete to prepare yourself for Pure Mathematics 10 or Applied Mathematics 10.

Mathematics Preparation 10

Module 1
Exploration of Numbers

Module 2
Number Connections

Module 3
Patterns and Algebra

Module 4
Polynomials

Module 5
Shape and Space

The document that you are now reading is called a Student Module Booklet. It has an accompanying Assignment Booklet.

You will find many visual cues or icons throughout this Student Module Booklet. Read the following statements to discover what each icon prompts you to do.

Use the Internet to explore a topic.

Use a scientific calculator.

Use the suggested answers in the Appendix to correct activities.

Answer the questions in the Assignment Booklet.

Throughout the course, you will be given instruction and practice on each of the following mathematical processes:

- connecting mathematical ideas
- developing and using estimation strategies
- developing and using mental math strategies
- using technology appropriately
- developing and using problem-solving strategies
- reasoning and justifying your answers
- using visualization to assist in processing information
- solving problems
- communicating mathematically

For example, to help you develop your problem-solving skills, several problem-solving strategies are explained in the Appendix of each Student Module Booklet, and you will be given several non-routine problems to solve in each module. Watch for the heading "Now Try This."

To help you develop your communication skills in mathematics, you will be asked to do several journal writings in each module. Watch for the heading "Looking Back."

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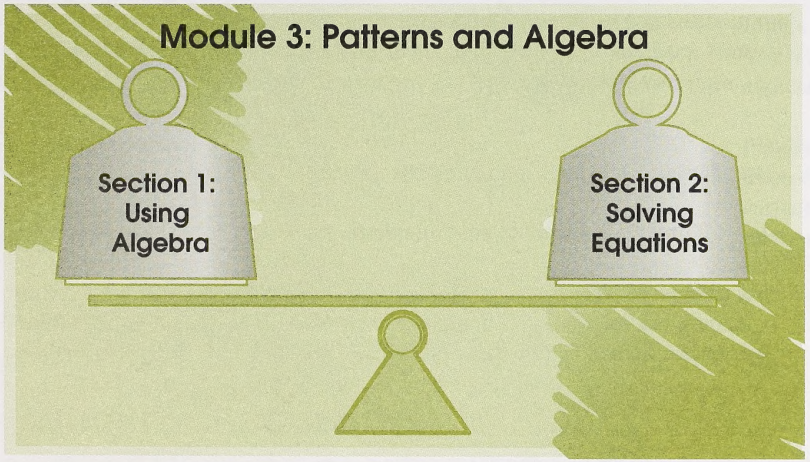
NOVA DEVELOPMENT CORPORATION

Module Overview

Think of a number between 1 and 10. Add 8 to it. Multiply the result by 2. Subtract 2. Divide by 2. Subtract the number that you first thought of. Is the answer 7? Try starting with a different number. Is the answer still 7?

If you use a symbol to represent the original number, you can discover why the answer to this problem is always the same. The use of symbols to represent numbers and the arithmetic done involving these symbols provide the basis of algebra.

In this module, you will extend your knowledge of algebra as you work with more complicated equations and use them to help solve problems. You will also use symbols to represent the volume and surface areas of three-dimensional shapes.

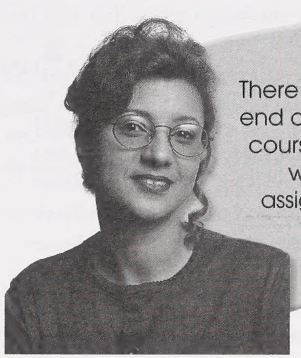


Assessment

Your mark for this module will be determined by how well you complete the assignments at the end of each section. In this module, you must complete two Assignment Booklets. The mark distribution is as follows:

Assignment Booklet 3A	
Section 1 Assignment	**25 marks**
Assignment Booklet 3B	
Section 2 Assignment	**45 marks**
Final Module Assignment	**30 marks**
TOTAL	**100 marks**

When doing the assignments, work slowly and carefully. You must do each assignment independently; but if you are having difficulties, you may review the appropriate section in this Student Module Booklet.



There is a supervised final test at the end of this course. Your mark for the course will be determined by how well you do on the module assignments and the supervised final test.

Stragegies for Completing the Module Successfully

To achieve success in this module, be sure to work slowly and systematically through the Student Module Booklet. Remember, your work in the Student Module Booklet will prepare you for your module assignments and final test.

Following are some strategies for completing the module successfully:

- Try to set realistic goals for each day; and once you've set these goals, stick to them.
- Read all the instructions in each Student Module Booklet carefully; answer all the questions in your mathematics binder; and check your answers by comparing your responses with the suggested answers in the Appendix.
- Keep a section of your mathematics binder for your journal entries. Get in the habit of describing new concepts, procedures, and strategies in your own words. Record useful ways to help you remember what a concept means. Make graphic organizers (such as context webs, Venn diagrams, and charts) to help you connect mathematical ideas.
- Ask someone who is taking Mathematics Preparation 10 to be your study partner. You will find that having a friend to discuss mathematics with will make your studying more enjoyable.
- If you need assistance from your teacher or your study partner, you may find it helpful to write your questions in the journal section of your mathematics binder before speaking to your teacher or study partner. This will help you pinpoint your problem.
- Take care when completing your module assignments. Be sure you have completed each part of the Assignment Booklets, and proofread the assignments before you submit them. Remember to review your module assignments after they are corrected; then file them in your binder so you can review your assignments and mathematics notes before you write the final test.

Good luck!

SECTION 1

Using Algebra

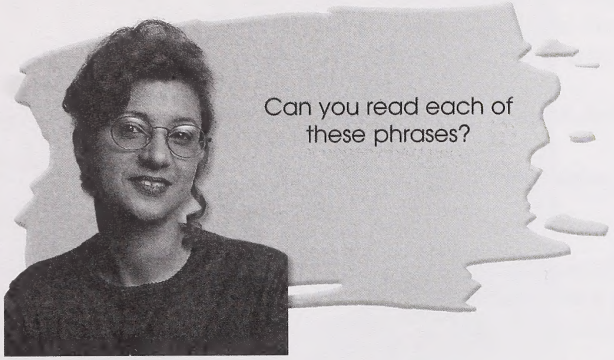
People often use birds and animals to represent ideas. Sometimes the animal represents the idea so well that it becomes a symbol. For example, the beaver is one of many symbols of Canada, an ant represents hard work, and a turtle represents slowness. What does an owl represent?

Mathematics is also rich in symbols. For example, 2 is the symbol for *two*, + is the symbol for *add* or *plus* or *positive*, and × is the symbol for *multiply*. Letters are also used as symbols to represent numbers that are unknown.

In this section, you will evaluate mathematical expressions and variable expressions. You will translate English phrases into mathematical expressions and variable expressions; and you will write equations and graph the information.

Finally, you will use algebra to determine the surface area and volume of three-dimensional shapes. You will analyse the relationships among the length, surface area, and volume of a figure. You will then use your understanding of these relationships to analyse designs in the packaging industry.

Activity 1: Translating Words and Phrases



deux et trois	dos y tres
zwei und drei	two plus three

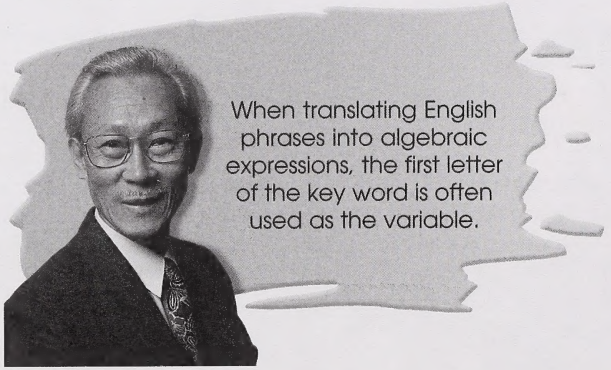
You probably cannot read every phrase because different languages have been used, but each phrase can be translated into the following mathematical expression:

$2+3$

Mathematics is a language made up of numerals and symbols that are understood by mathematicians all over the world, no matter which language is spoken.

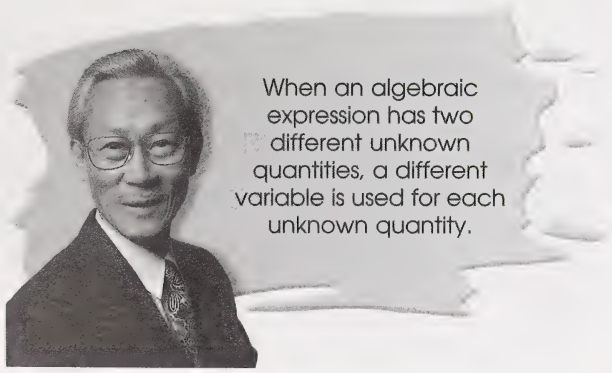
Because non-mathematical languages, such as English, are very complex, a mathematical expression can be translated into many different words and phrases. For example, "$2+3$" can be translated into each of the following English phrases:

- two plus three
- two increased by three
- the sum of two and three
- three more than two
- three added to two



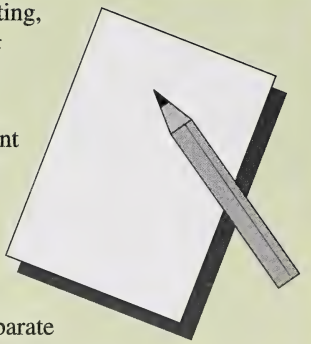
Here are some examples where the first letter of the key word is used as the variable.

Phrase	Algebraic Expression
5 times Jean's **a**ge	$5a$
the **d**istance from Albert's house to work divided by 2	$\frac{d}{2}$
Helen's **m**ass decreased by 2 kilograms	$m-2$
10 dollars more than Rachel's **w**ages	$w+10$
the sum of the **l**ength and **w**idth of a house	$\ell+w$
the number of **p**atients times the **c**ost per patient	pc
the **p**rice minus the **d**iscount	$p-d$



When an algebraic expression has two different unknown quantities, a different variable is used for each unknown quantity.

Remember the following points when translating English phrases into algebraic expressions:



- Get in the habit of writing, rather than printing, variables. Be careful to make the variable x different from the multiplication sign ($\times$). Make the variable ℓ different from the number 1. Make the variable o or O different from the number 0.
- Most mathematicians do not include the multiplication sign between a number and a variable, but you may occasionally see brackets or the multiplication dot to separate factors. For example, you may see the phrase "five times Jean's age" translated as $5a$, $5(a)$, or $5 \cdot a$.
- Most mathematicians use a bar to show division, but you may occasionally see the division sign $(\div)$ used. For example, you may see the phrase "the distance from Albert's house to work divided by 2" translated as $\frac{d}{2}$ or $d \div 2$.

The order of operations is important when working with algebraic expressions. The use of parentheses helps show which operations need to be completed first.

The expression $3n+4$ is read as "three times n, plus four" or "four more than three times n."

The expression $3(n+4)$ is read as "three times the sum of n and four."

1. Write each of the following variable expressions in words.

a. $2n-1$ **b.** $2(n-1)$ **c.** $4+5n$

d. $(4+5)n$ **e.** $6-3n$ **f.** $(6-3)n$

2. Translate each of the following phrases into variable expressions. Let n be the number.

a. three more than the product of two and a number

b. four times a number, decreased by five

c. two times the sum of a number and five

d. one less than a number, times four

3. Translate the following phrases into algebraic expressions.

a. Marco's age increased by two years

b. twice the distance from Muriel's house to school

c. five times the price of a car

d. a boy's age twelve years from now

e. two-thirds of the students

f. Katya's earnings divided by two

g. three times the sum of Angela's and Jayden's masses

h. the number of words typed in a minute minus the number of errors

i. one-half of Rebecca's height, plus twice Mitchell's height

Check your answers by turning to the Appendix, page 99.

Looking Back

In this activity, you translated English phrases into mathematical expressions and vice versa. You will practise this skill throughout this module as you solve problems.

4. Copy and complete the following chart into your journal, inserting all of the words and phrases you know that are represented by each operation. Leave room in the right column for each symbol to add more words and phrases as you encounter them.

Operation Symbol	Words and Phrases
+	
−	
×	
÷	

Check your answers by turning to the Appendix, page 100.

Activity 2: Formulas and Graphing

Did you know that there is a simple way of estimating how far away a thunderstorm is?

Sound travels about one-third of a kilometre per second. Therefore, the distance you are from a thunderstorm can be estimated by counting the number of seconds between seeing the lightning and hearing the resulting thunder and dividing this number by 3.

This rule can be expressed by the following formula, where d is the distance (in kilometres) from the storm and t is the time (in seconds) between seeing the lightning and hearing the thunder.

$$d = \frac{t}{3}$$

A formula is an equation showing how one quantity is related to one or more other quantities. A formula is usually written using variables.

Three examples of how far away a storm is are as follows:

- How far away is the storm if you can count 3 s between seeing the lightning and hearing the thunder?

  If $t = 3$, then

  $$\begin{aligned} d &= \frac{t}{3} \\ &= \frac{3}{3} \\ &= 1 \end{aligned}$$

  The storm is about 1 km away.

- How far away is the storm if you count 6 s between seeing the lightning and hearing the thunder?

  If $t = 6$, then

  $$\begin{aligned} d &= \frac{t}{3} \\ &= \frac{6}{3} \\ &= 2 \end{aligned}$$

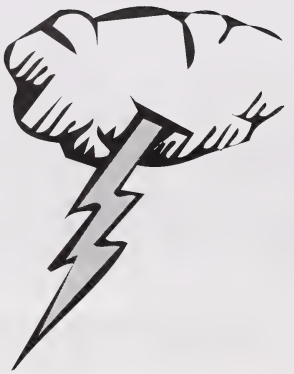
  The storm is about 2 km away.

- How far away is the storm if you count 9 s between seeing the lightning and hearing the thunder?

  If $t = 9$, then

  $$\begin{aligned} d &= \frac{t}{3} \\ &= \frac{9}{3} \\ &= 3 \end{aligned}$$

  The storm is about 3 km away.



The formula $d = \frac{t}{3}$ expresses the relationship between time and distance. This relationship can also be shown in the following ways:

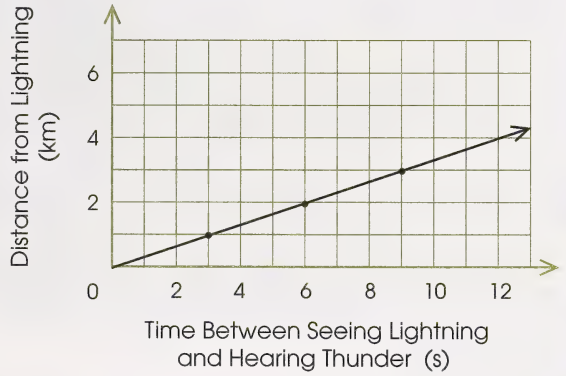
- using a table

Time Between Seeing Lightning and Hearing Thunder (s)	Distance from Lightning (km)
3	1
6	2
9	3
...	...

- using ordered pairs

$(3, 1),\ (6, 2),\ (9, 3),\ \ldots$

- using a graph



For the preceding graph, think of time as the independent variable and distance as the dependent variable.

In a relation, the independent variable is the variable you control and the dependent variable is the variable that depends on the values chosen for the independent variable.

When graphing, place the independent variable on the horizontal axis and the dependent variable on the vertical axis.

1. Stephan's Ski Shoppe rents skis. The cost of renting skis is $12.00, plus $0.50 per hour.

This rule can be expressed by the following formula, where C is the cost (in dollars) and t is the time (in hours).

$$C = 12.00 + 0.50t$$

a. What is the cost of renting skis for 1 h?

b. What is the cost of renting skis for 2 h?

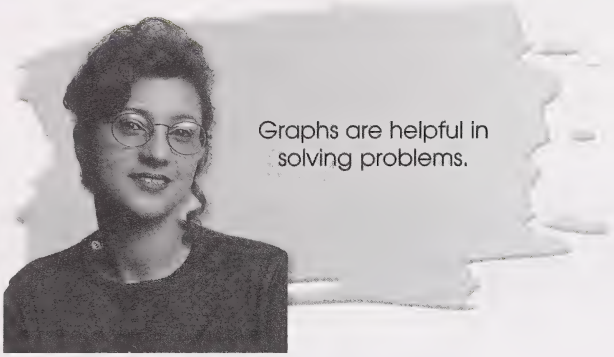
c. What is the cost of renting skis for 3 h?

d. What is the cost of renting skis for 4 h?

e. Make a graph to show the relationship between the rental time and cost of renting skis.

f. How does the rental time affect the cost of renting the skis?

Check your answers by turning to the Appendix, page 100.



Example

Dexter and Liliana are getting married next summer and have gone to Best Printing to get a quote on the invitations. The sales person gives Dexter and Liliana two options for the cost of printing invitations.

Option 1	*Option 2*
$15.00, plus $0.10 per invitation	*$0.15 per invitation*

For what number of invitations is Option 1 less expensive?
For what number of invitations is Option 2 less expensive?

Solution

Step 1: Represent the unknown quantities by variables.

Let C be the cost (in dollars) of printing.
Let n be the number of invitations printed.

Step 2: Write a formula for Option 1.

$$C = 15.00 + 0.10n$$

Step 3: Write a formula for Option 2.

$$C = 0.15n$$

Step 4: Make a table of values for Option 1.

Option 1

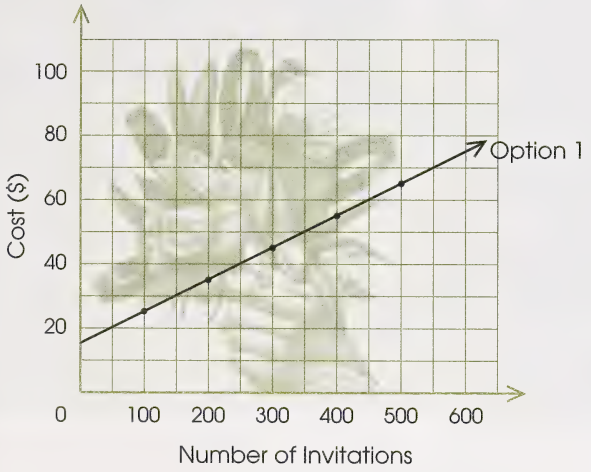
Number of Invitations	Cost ($)
100	25
200	35
300	45
400	55
500	65

Step 5: Make a table of values for Option 2.

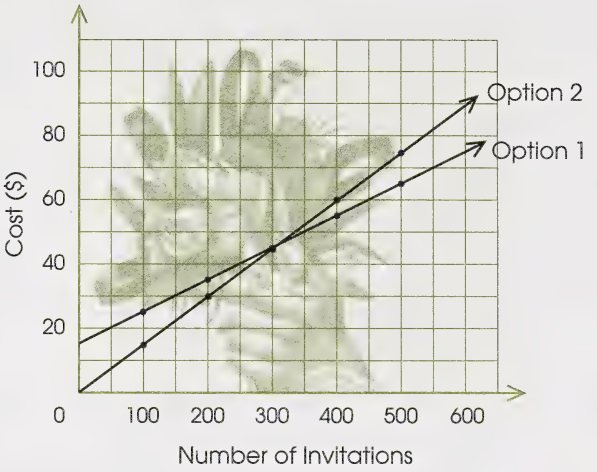
Option 2

Number of Invitations	Cost ($)
100	15
200	30
300	45
400	60
500	75

Step 6: Make a graph of Option 1. Label the graph.



Step 7: Make a graph of Option 2. Place it on the same grid as Option 1.



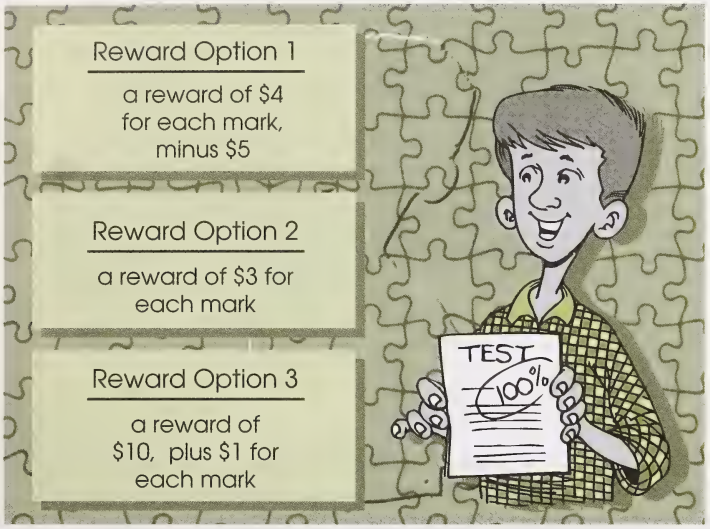
Step 8: Compare the graphs and answer the problem.

Option 1 is less expensive if Dexter and Liliana order more than 300 invitations.

Option 2 is less expensive if Dexter and Liliana order fewer than 300 invitations.

Note: Option 1 and Option 2 cost the same when 300 invitations are ordered.

2. Nicholas's mom wants to encourage Nicholas to study for a test. She gives him a choice of three rewards.



There will be 10 questions on the test, where each correct answer is worth 1 mark. (Partial marks can be given.)

a. Write a formula for each reward option.

b. Make a table of values for each reward option.

c. Use the tables of values to graph each reward option on the same grid.

d. Which option should Nicholas pick if he expects to earn no marks? Why?

e. Which option should Nicholas pick if he expects to get all ten questions correct?

f. Which option should Nicholas pick if he expects to earn five out of ten marks?

Check your answers by turning to the Appendix, page 101.

In order to observe patterns in data, it is important that data is arranged in a systematic way.

3. Ron wondered if the cooking time for a roast is related to the mass of the roast; so, he conducted an experiment. Ron cooked several roasts of different size and noted the cooking time for each. Here are his results.

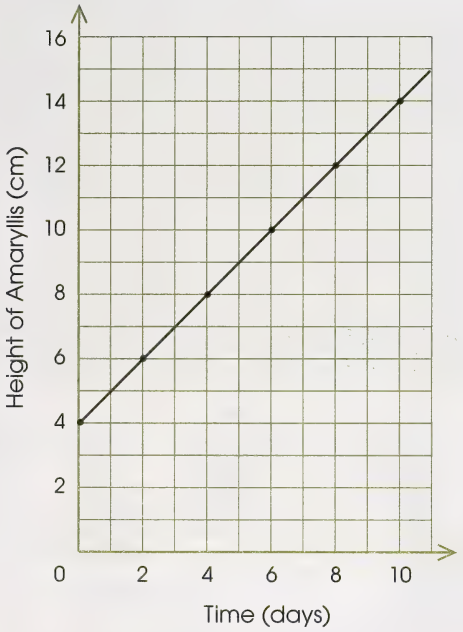
Mass of Roast (kg)	Cooking Time (min)
3	120
5	200
2	80
1	40
4	160

a. Ron was unable to find a pattern because the data was disorganized. Rearrange the data so that the roasts are listed in order of mass.

b. Write a formula to describe how the cooking time of a roast is related to its mass. Let c be the cooking time (in minutes), and let m be the mass (in kilograms).

c. Use the formula to calculate the cooking time for a 4.5-kg roast.

4. Noel wondered if there is a relationship between the height of an amaryllis plant and the length of the growing time. She planted an amaryllis bulb, measured the height of the plant at regular intervals, and drew the following graph.



a. Why didn't Noel put an arrowhead on her graph?

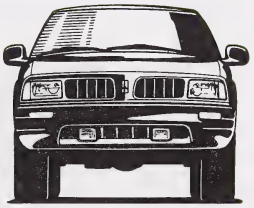
b. Use the graph to complete the following table. **Note:** The bulb has a height as soon as it is planted because the bulb is planted with its top third above ground.

Time (days)	Height (cm)
0	
1	
2	
3	
4	
5	

c. Write a formula to describe how the height of an amaryllis is related to its growing time. Let h be the height (in centimetres), and let t be the time (in days).

d. Identify the independent variable and the dependent variable. Explain how you arrived at your answer.

5. When travelling by car at night, a person can be "blinded" by the headlights of an oncoming vehicle. Jorge wondered how far an average person travelled while "blind." He discovered the distance varied and that the distance was related to the speed of the car.



The following table gives data on driving while "blind."

Speed of Car (km/h)	Distance Travelled While "Blind" (m)
0	0
30	25
60	50
90	75
120	100

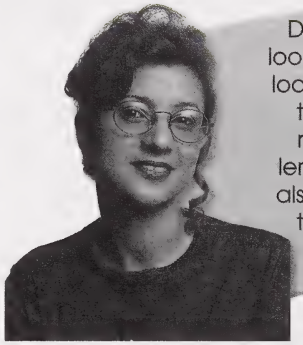
a. Make a graph of the data.

b. Write a formula describing how the distance travelled while "blind" is related to the speed of the car. Let s represent the speed (in km/h), and let d represent the distance (in metres).

c. Use the formula from question 5.b. to predict how far a person will travel while "blind" if the car is travelling at a speed of 96 km/h.

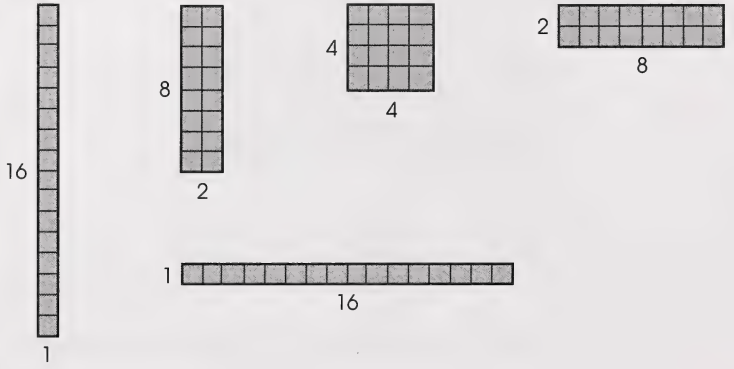
Check your answers by turning to the Appendix, page 103.

Formulas Relating Perimeter and Area to Length



Determining formulas involves looking for patterns. Now, you will look for patterns to discover how the perimeter and area of a rectangle are related to the length of the rectangle. You will also use graphs to discover how the perimeter and area of a rectangle are related.

6. There are many rectangles with an area of 16 square units. Here are some of the possibilities.



a. Use the given rectangles to complete the following table.

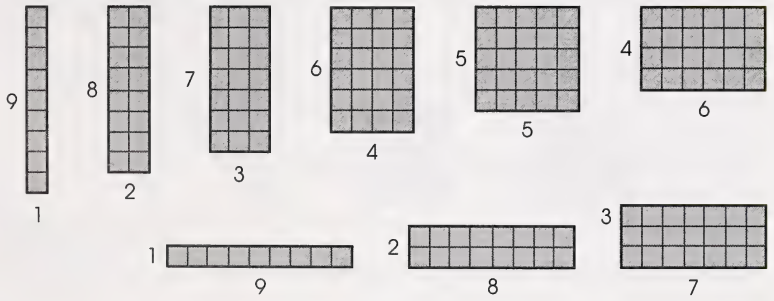
Rectangles with an Area of 16 Square Units

Length (units)	Width (units)	Perimeter (units)
1	16	
2	8	
4	4	
8	2	
16	1	

b. Which rectangle or rectangles have the greatest perimeter?

c. Which rectangle or rectangles have the least perimeter?

7. There are many rectangles with a perimeter of 20 units. Here are some of the possiblilities.



a. Use the given rectangles to complete the following table.

Rectangles with a Perimeter of 20 Units

Length (units)	Width (units)	Area (square units)
1	9	
2	8	
3	7	
4	6	
5	5	
6	4	
7	3	
8	2	
9	1	

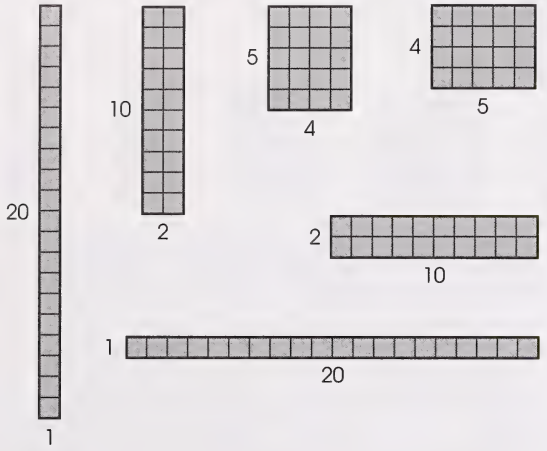
b. Graph the set of ordered pairs (length, area) for the rectangles from the table in question 7.a.

c. Because a rectangle with a perimeter of 20 units can have fractional units for its length, connect the dots on the graph in question 7.b.

d. What does the graph tell you about the relationship between length and area?

e. What do you notice about the rectangle with the greatest area?

8. There are many rectangles with an area of 20 square units. Here are some possibilities.



a. Use the given rectangles to complete the following table.

Rectangles with an Area of 20 Square Units

Length (units)	Width (units)	Perimeter (units)
1	20	
2	10	
4	5	
5	4	
10	2	
20	1	

b. Graph the set of ordered pairs (length, perimeter) for the rectangles from the table in question 8.a.

c. Because a rectangle with an area of 20 square units can have fractional units for its length, connect the dots on your graph in question 8.b.

d. What does the graph tell you about the relationship between length and perimeter?

9. Optek has decided to build a rectangular dog run. He wants to use as little fencing as possible. If his dog needs about 144 m^2, how long and wide should the dog run be?

10. Arlene wants to build a rectangular pen for her chickens. The chickens need an area of 36 m^2. To save on fencing, she plans to use a river that runs through her farm as one side of the pen. She also wants to use whole numbers for her measurements.

a. Draw and label all the possible designs for the pen.

b. Which is the most economical design for the pen?

c. Which is the most expensive design for the pens?

11. A hardware store sells fencing by the metre. If Mr. Petruk purchases 42 m of fencing to build a rectangular pen, what will the dimensions of the pen be if the length is 1 m more than the width?

12. Suppose you have 16 m of fencing. Design the largest pen possible with this amount of fencing. Your design can be any shape.

Check your answers by turning to the Appendix, page 104.

Looking Back

In this activity, you wrote formulas describing how quantities are related. You evaluated these formulas for certain values and graphed the ordered pairs on a grid. You also related perimeter to area.

In your journal, complete the following sentences.

13. a. If rectangles with the same area are being compared, the rectangle with the smallest perimeter would…

b. If rectangles with the same perimeter are being compared, the rectangle with the largest area would…

14. At a certain car-rental outlet, rental costs are $45.00 per day plus $0.25 per kilometre.

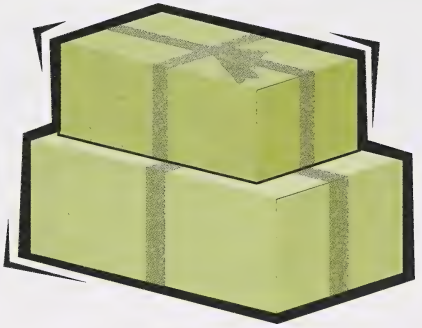
a. Determine a formula, defining the variables, that describes the cost of renting a car from this rental outlet.

b. What does the cost of renting a car from this outlet depend on?

Check your answers by turning to the Appendix, page 109.

Activity 3: Surface Area

Estimating the amount of wrapping paper needed to gift-wrap a present can sometimes be a problem. Which of the following presents do you think required more wrapping paper?



The amount of wrapping paper is related to the surface area of the container being wrapped.

In this activity, you will calculate the surface area of three-dimensional (3-D) shapes.

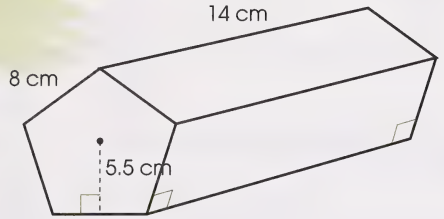
The total surface area of a 3-D shape is the sum of the areas of all its flat surfaces (faces) and curved surfaces.

It is helpful to use a net to visualize the faces.

A net is a two-dimensional (2-D) figure that can be assembled to create a three-dimensional shape.

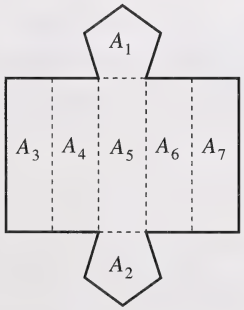
Example

Find the surface area, S, of the right pentagonal prism given. **Hint:** Each pentagonal face is a regular polygon.



Solution

Step 1: Represent the surface area of the regular pentagonal prism as the sum of the areas of its seven faces (as shown in the following net).



Note: $A_1 = A_2$ and $A_3 = A_4 = A_5 = A_6 = A_7$

Step 2: Calculate the area of the two end faces, A_1 and A_2. **Note:** You may refer to the Formula Sheet given in the Appendix. The end faces are regular pentagons; so, find the formula for a regular pentagon.

$$A = \frac{nsa}{2}$$
$$= \frac{5 \times 8 \times 5.5}{2}$$
$$= 110$$

Each end face has an area of $110\ \text{cm}^2$. Thus, the two end faces have an area of $2 \times 110 = 220\ \text{cm}^2$.

Step 3: Find the area of each of the side faces.

$$A = \ell w$$
$$= (14)(8)$$
$$= 112$$

Each rectangular face has an area of $112\ \text{cm}^2$. Thus, the five side faces have an area of $5 \times 112 = 560\ \text{cm}^2$.

Step 4: Find the surface area.

$$S = 220 + 560$$
$$= 780$$

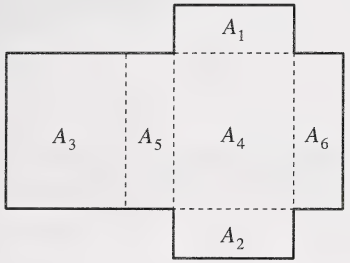
The total surface area is $780\ \text{cm}^2$.

Example

Find the surface area, S, of the cereal box.

Solution

Step 1: Represent the surface area of the box as the sum of the areas of its six faces. This is shown in the following net.



Note: $A_1 = A_2$, $A_3 = A_4$, and $A_5 = A_6$

Step 2: Calculate the area of the top and bottom faces, A_1 and A_2.

Note: Each rectangle is $19\ \text{cm} \times 5\ \text{cm}$.

$$\begin{aligned} A_1 &= \ell w \\ &= 19(5) \\ &= 95 \qquad \therefore A_2 = 95 \end{aligned}$$

The area of A_1 and A_2 together is $2 \times 95 = 190\ \text{cm}^2$.

Step 3: Calculate the area of the front and back faces, A_3 and A_4.

Note: Each rectangle is $25\ \text{cm} \times 19\ \text{cm}$.

$$\begin{aligned} A_3 &= \ell w \\ &= 25(19) \\ &= 475 \qquad \therefore A_4 = 475 \end{aligned}$$

The area of A_3 and A_4 together is $2 \times 475 = 950\ \text{cm}^2$.

Step 4: Calculate the area of the two side faces, A_5 and A_6.

Note: Each rectangle is $25\ \text{cm} \times 5\ \text{cm}$.

$$\begin{aligned} A_5 &= \ell w \\ &= 25(5) \\ &= 125 \qquad \therefore A_6 = 125 \end{aligned}$$

The area of A_5 and A_6 together is $2 \times 125 = 250\ \text{cm}^2$.

Step 5: Find the surface area of the box.

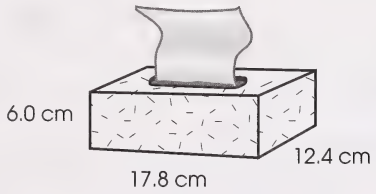
$$\begin{aligned} S &= (A_1 + A_2) + (A_3 + A_4) + (A_5 + A_6) \\ &= 190 + 950 + 250 \\ &= 1390 \end{aligned}$$

The surface area of the box is $1390\ \text{cm}^2$.

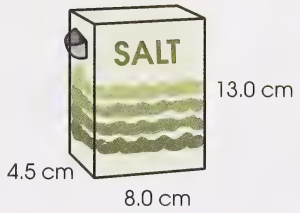
Use a scientific calculator to complete problems 1, 2, and 3.

1. Find the surface area of each of the following containers.

a.



b.



2. The following chocolate box is a triangular prism. The length of the box is 16.7 cm. The triangular base has sides of 3.0 cm and a height of 2.6 cm. What is the surface area of the box? Round your answer to the nearest tenth.

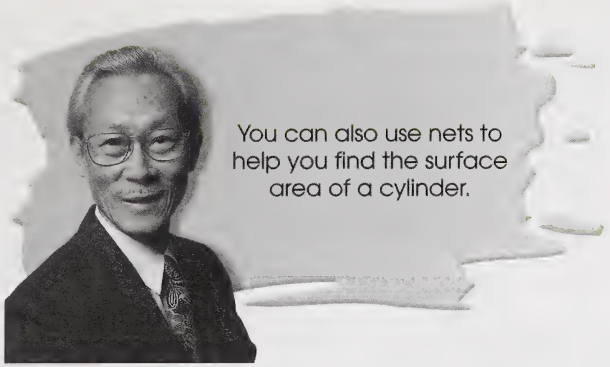
[1]

[1] © Kraft Foods Canada, Inc. All rights reserved.

3. The box in the photograph is a prism. The base of the box is a regular hexagon. Each side of the base is 7.5 cm. The apothem of the base is 6.5 cm. The height of the box is 5.8 cm. What is the surface area of the box?

Remember: The apothem is the perpendicular distance from the centre of a polygon to any one of its sides.

Check your answers by turning to the Appendix, page 109.

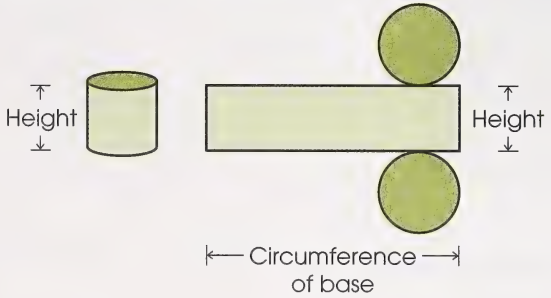


Example

Find the surface area of a cylinder with a diameter of 10 cm and a height of 32 cm. Round your answer to the nearest whole number.

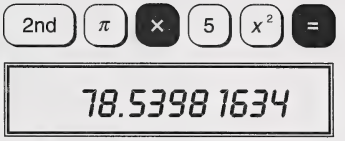
Solution

Step 1: Represent the surface area of the cylinder as the sum of the areas of its three faces (as shown in the following net). **Note:** $A_1 = A_2$



Step 2: Find the area of A_1 and A_2. **Note:** Use the π key on your calculator.

$$A_1 = \pi r^2$$
$$= \pi(5)^2$$
$$\doteq 78.5$$

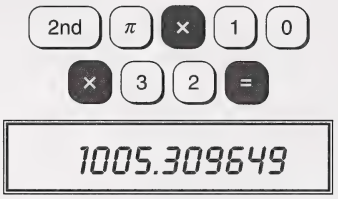


$\therefore A_2 \doteq 78.5$ and $A_1 + A_2 \doteq 157.0$

The area of A_1 and A_2 together is about $157.0\ \text{cm}^2$.

Step 3: Find the area of curved surface. **Note:** The curved surface is a rectangle when it is opened and flattened. (Imagine taking a label off a can). The length of the rectangle is equal to the circumference, *C*, of the circular base. The width of the rectangle is equal to the height, *H*, of the cylinder.

$$\therefore A_3 = \ell w$$
$$= CH$$
$$= (\pi d)(32)$$
$$= \pi(10)(32)$$
$$\doteq 1005.3$$



The area of A_3 (the curved surface) is about $1005.3\ \text{cm}^2$.

Step 4: Find the surface area of the cylinder, rounding to the nearest whole number.

$$\begin{aligned} s &= (A_1 + A_2) + A_3 \\ &= 157.0 + 1005.3 \\ &\doteq 1162.3 \\ &\doteq 1162 \end{aligned}$$

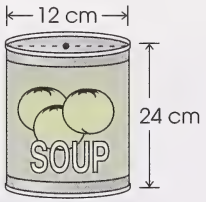
The area of the cylinder is about $1162\ \text{cm}^2$.

Because the preceding example was a multi-step problem, the answers in each part were rounded to one decimal place greater than required to avoid rounding errors.

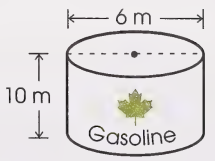
Use a scientific calculator to complete problems 4 and 5.

4. Find the surface area of the following cylinders. Round your answers to the nearest whole number.

a.

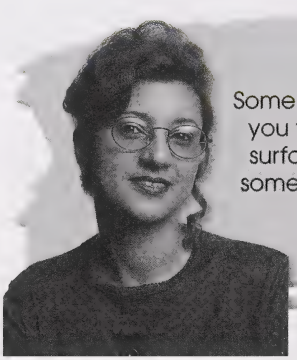


b.



5. A 340-mL can of pop approximates a cylinder. If a can of pop has a diameter of about 6 cm and a height of 12 cm, what is its surface area? Round your answer to the nearest whole number.

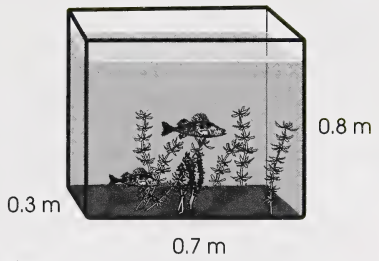
Check your answers by turning to the Appendix, page 112.



Some problems require you to calculate the surface area of only some faces or curved surfaces.

Example

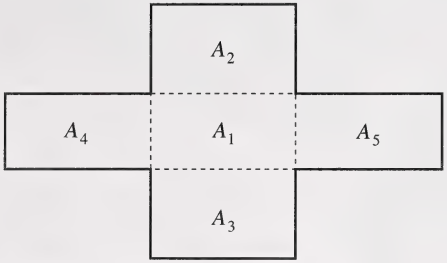
What is the surface area of the exterior of this aquarium?



Solution

Step 1: Represent the surface area of the exterior of the aquarium as the sum of the areas of five faces (bottom, back, front, two sides/ends).

Note: $A_2 = A_3$ and $A_4 = A_5$



Step 2: Calculate the area of the bottom, A_1. **Note:** The rectangle is $0.7\text{ m} \times 0.3\text{ m}$.

$$\begin{aligned} A_1 &= \ell w \\ &= 0.7(0.3) \\ &= 0.21 \end{aligned}$$

The area of A_1 is 0.21 m^2.

Step 3: Calculate the area of the front and back, A_2 and A_3. **Note:** Each rectangle is $0.7\text{ m} \times 0.8\text{ m}$.

$$\begin{aligned} A_2 &= \ell w \\ &= 0.7(0.8) \\ &= 0.56 \end{aligned} \qquad \therefore A_3 = 0.56$$

The area of A_2 and A_3 together is 1.12 m^2.

Step 4: Calculate the area of the two sides/ends, A_4 and A_5. **Note:** Each rectangle is $0.8\text{ m} \times 0.3\text{ m}$.

$$\begin{aligned} A_4 &= \ell w \\ &= 0.8(0.3) \\ &= 0.24 \end{aligned} \qquad \therefore A_5 = 0.24$$

The area of A_4 and A_5 together is 0.48 m^2.

Step 5: Calculate the surface area of the exterior of the aquarium.

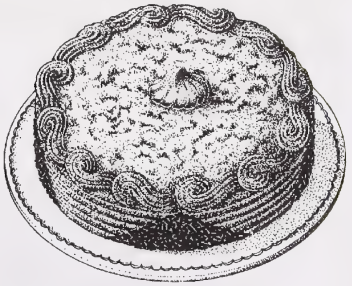
$$\begin{aligned} S &= A_1 + (A_2 + A_3) + (A_4 + A_5) \\ &= 0.21 + 1.12 + 0.48 \\ &= 1.81 \end{aligned}$$

The surface area of the exterior of the aquarium is 1.81 m^2.

Use a scientific calculator to complete problems 6, 7, and 8.

6. A cylindrical pool has a plastic liner covering its interior walls and floor. If the pool has a diameter of 4 m and a depth of 2 m, what is the surface area of the plastic liner? Round your answer to the nearest tenth.

7. A cake has a diameter of 20 cm and a height of 13 cm.



a. If the top and sides of the cake are covered with icing, what is the area of the iced surfaces of the cake? Round your answer to the nearest whole number.

b. The cake is cut into two identical layers. If the top and sides of each layer are covered with icing, what is the area of the iced surfaces of the cake? Explain the reasoning you use to find the new area. Round your answer to the nearest whole number.

8. A rectangular room is 8 m long and 4 m wide, and the walls are 3 m high. If 1 pail of paint covers 12 m^2, how much paint is needed to paint the ceiling and walls of the room? **Hint:** Do not consider windows and doors.

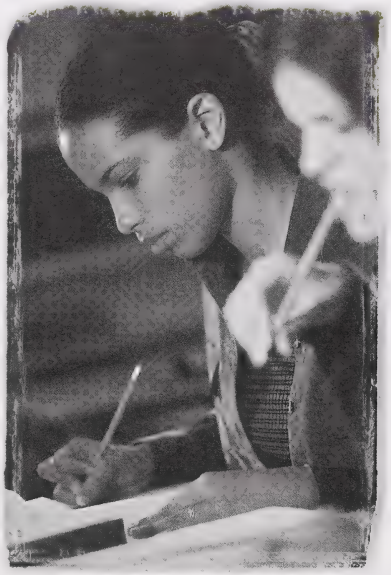
Check your answers by turning to the Appendix, page 114.

Looking Back

In this section, you determined the surface areas of right prisms and cylinders. You discovered that using a net for each figure is useful when representing the surface area as a sum of areas.

9. In your journal, sketch a triangular prism, rectangular prism, and cylinder. Beside each sketch, describe the surface area as a sum of areas and the measurements necessary to calculate the total surface area?

Check your answers by turning to the Appendix, page 117.



Activity 4: Volume

In the moving business, the amount of space used is very important. Movers are very skilled at arranging the contents of the moving van so that the boxes take up the least amount of space possible.

The amount of space an object occupies is called its volume.

Volume of a Prism and a Cylinder

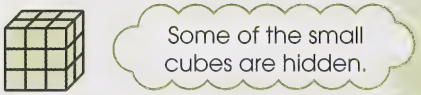
You can find the volume of a rectangular prism by counting the number of cubic units.

Example

What is the volume of a rectangular prism that is 3 units long, 2 units wide, and 3 units high?

Solution

Use cubes to build a rectangular prism that is 3 units by 2 units by 3 units.

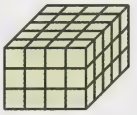


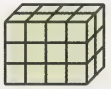
The volume of this rectangular prism is 18 cubic units.

Complete questions 1, 2, and 3, dealing with volumes of rectangular prisms.

1. Find the volume (in cubic units) of each of the following rectangular prisms. If you wish, build the rectangular prisms with sugar cubes or blocks.

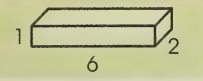
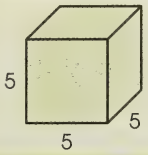
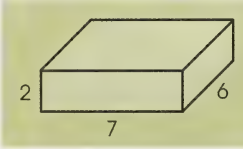
a.

b. 

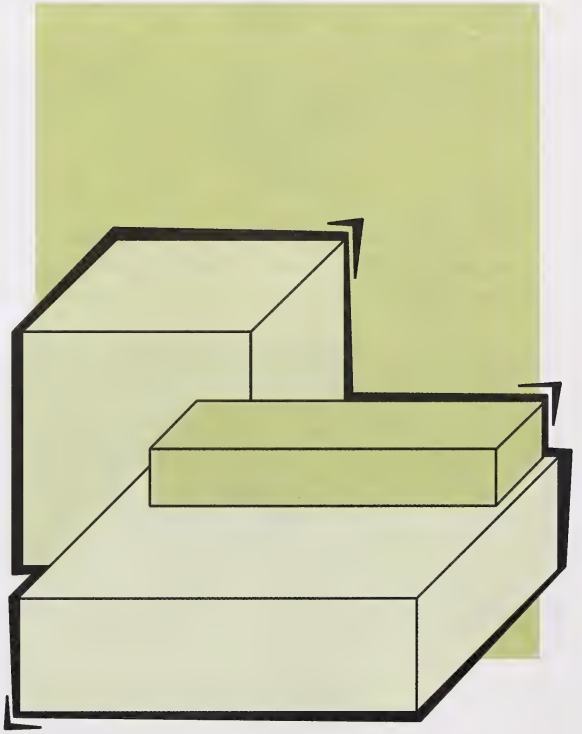
c. 

d.

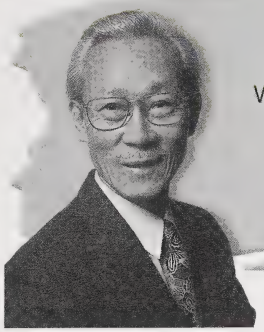
2. The following table gives the lengths, widths, heights, and volumes of various rectangular prisms. What pattern do you notice? How is the volume of a rectangular prism related to the length, width, and height.

Rectangular Prisms (measured in units)	Length (units)	Width (units)	Height (units)	Volume (cubic units)	
	6	2	1	12	$6\times2\times1$
	5	5	5	125	$5\times5\times5$
	7	6	2	84	$7\times6\times2$

3. Explain why the volume of a rectangular prism can be expressed as the formula $V = \ell wh$, where V is the volume, ℓ is the length, w is the width, and h is the height of the rectangular prism.



Check your answers by turning to the Appendix, page 117.



What pattern did you discover about volume?

I discovered that the volume of a rectangular prism is equal to the area of the base times the height.

This relationship for the volume of a prism can be expressed by the formula $V = BH$, where V is the volume of any prism, B is the area of the base of the prism or cylinder, and H is the height of the prism or cylinder.

This formula can also be used to find the area of a cylinder. **Note:** Upper case letters are often used for the area of the base and the height of the prism or cylinder to distinguish them from the variables used in area formulas.

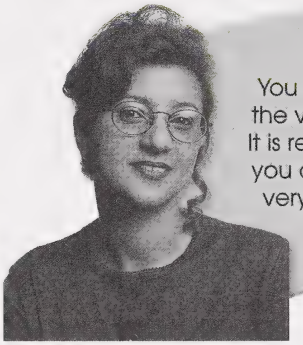
4. a. What does b represent in the formulas $A = bh$, $A = \frac{1}{2}bh$, and $A = \frac{(b_1 + b_2)h}{2}$?

b. What does h represent in the formulas $A = bh$, $A = \frac{1}{2}bh$, and $A = \frac{(b_1 + b_2)h}{2}$?

c. What does B represent in the formula $V = BH$?

d. What does H represent in the formula $V = BH$?

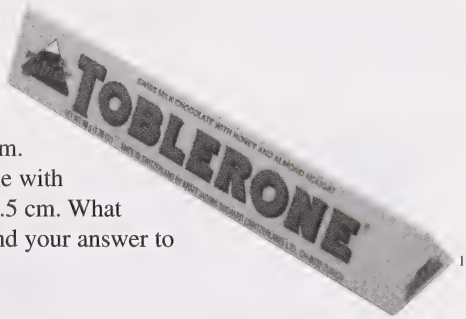
Check your answers by turning to the Appendix, page 117.



You can use a calculator to find the volume of a prism or cylinder. It is recommended, however, that you also estimate the answer; it is very easy to press a wrong key.

Some people prefer to calculate the answer first and then use estimation to determine if the answer is reasonable. Others prefer to estimate the answer first.

Example

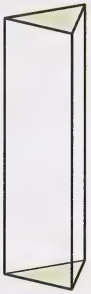


This chocolate box is in the shape of a triangular prism. The length of the box is 16.7 cm. The base of the box is a triangle with sides of 3 cm and a height of 2.5 cm. What is the volume of the box? Round your answer to the nearest tenth.

[1]

Solution

Step 1: Calculate the area of the base, B. **Note:** The base is a triangle.



$$B = \frac{1}{2}bh$$
$$= \frac{1}{2}(3)(2.5)$$
$$= 3.75$$

The area of the base is $3.75\ \text{cm}^2$.

Step 2: Calculate the volume, V, of the prism. **Note:** The box sits on its triangular base; you can picture that the "length" of the box becomes its "height."

$$V = BH$$
$$= 3.75(16.7)$$
$$\doteq 62.6$$

The volume of the box is about $62.6\ \text{cm}^3$.

Use estimation to determine if the answer is reasonable.

Rounding

$$B = \frac{bh}{2} \qquad V = BH$$
$$\doteq \frac{3(3)}{2} \qquad \doteq 4.5(20)$$
$$\doteq 4.5 \qquad \doteq 90$$

Front-end Digits

$$B = \frac{bh}{2} \qquad V = BH$$
$$\doteq \frac{3(2)}{2} \qquad \doteq 3(16)$$
$$\doteq 3 \qquad \doteq 48$$

The answer is reasonable.

Example

Jonathan is measuring the dimensions of a cylindrical bottle. The diameter of the base is 6 cm. The height of the bottle is 8 cm. What is the volume of the bottle? Round your answer to the nearest tenth.

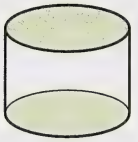
[1] © Kraft Foods Canada, Inc. All rights reserved.

Solution

Step 1: Calculate the area of the base, B. **Note:** The base is a circle with a diameter of 6 cm.

$$B = \pi r^2$$
$$= \pi(3)^2$$
$$\doteq 28.27$$

$d = 6$ cm
$r = 3$ cm



The area of the base is about $28.27\ \text{cm}^2$.

Step 2: Calculate the volume, V, of the cylinder.

$$V = BH$$
$$\doteq 28.27(8)$$
$$\doteq 226.2$$

The volume of the bottle is about $226.2\ \text{cm}^3$.

Use estimation to determine if the answer is reasonable.

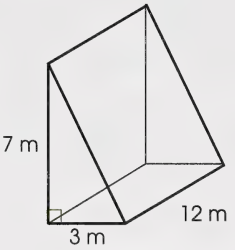
Rounding

$$B = \pi r^2 \qquad V = BH$$
$$\doteq 3(3^2) \qquad \doteq 30(8)$$
$$\doteq 3(9) \qquad \doteq 240$$
$$\doteq 27$$
$$\doteq 30$$

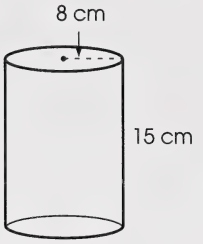
The answer is reasonable.

5. Calculate the volume of each of the following objects. Round your answers to the nearest tenth. **Note:** Estimate to determine if your answers are reasonable.

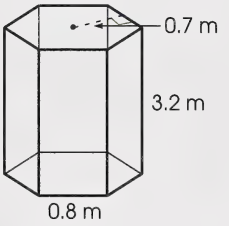
a.



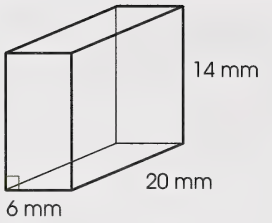
b.



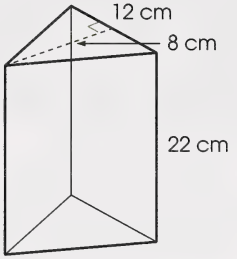
c.



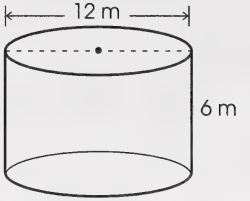
d.



e.



f.



6. The box in the photograph is a prism. The base of the prism is a decagon. Each side of the base is 5.0 cm. The apothem of the base is 7.7 cm. The height of the box is 5.8 cm. What is the volume of the box?

7. A Canadian $1 coin (a loonie) is a prism. The base of the coin has 11 sides, each 8.0 mm in length. The apothem of the base is 13.3 mm. The height of the coin is 2.0 mm. What is the volume of a loonie? Round your answer to the nearest whole number.

8. How much wax is needed to make a cylindrical candle with a radius of 5 cm and a height of 12 cm? Round your answer to the nearest whole number.

Check your answers by turning to the Appendix, page 118.

Volume of a Pyramid

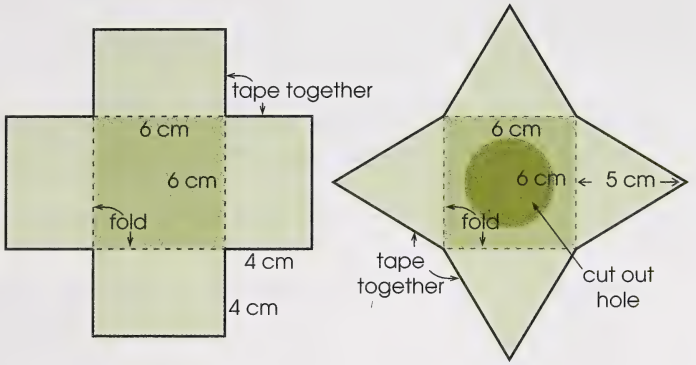
Humidity control is very important for the healthy growth of plants in the Muttart Conservatory in Edmonton. The ventilation system of the tropical pavilion must exchange the entire volume of air in the pyramid every half hour. To choose the appropriate ventilation system, the contractor must know the volume of air in the pyramid.

What is the volume of air in one of the pyramids at the Muttart Conservatory? A formula that can be used to calculate the volume would make this task much easier.

You know that volume is measured in cubic units and that the volume of a rectangular prism is found using the formula $V = BH$, where B is the area of the base and H is the height.

See if you can discover a formula for finding the volume of a pyramid as you work through the following investigation.

Step 1: Cut out the nets given at the back of the Appendix, or make your own nets for a rectangular prism and a rectangular pyramid like the ones shown. Glue the page with the nets to construction paper or boxboard (such as a shoe box or cereal box).



Step 2: Fold and tape all of the sides of the net on the left together to make an open-topped rectangular prism.

Step 3: Cut out the circle in the middle of the net for the rectangular pyramid. Then fold and tape all of the sides together.

Step 4: Fill the pyramid with rice or any other small-grained, pourable material, carefully supporting the pyramid so it does not come apart. Now, pour the rice from the pyramid into the rectangular prism.

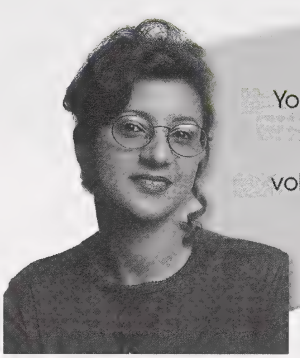
9. Complete the following to arrive at the formula for the volume of a pyramid.

a. The rectangular prism and rectangular pyramid have the same base. Are the heights the same?

b. How many times must you do Step 4 in order to completely fill the rectangular prism? What does this tell you about the relationship between the volume of a prism and the volume of a pyramid that have the same base and the same height?

c. Write a formula for the volume of a rectangular pyramid based on your findings.

Check your answers by turning to the Appendix, page 121.



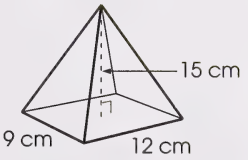
You have discovered that the volume of a rectangular pyramid is one-third the volume of a rectangular prism with the same base and height.

You could conduct other experiments with prisms and pyramids with bases shaped like triangles (or any regular polygon) and find that the results are the same.

The volume of any right pyramid is found by using the general formula $V = \frac{1}{3}BH$ ($\frac{1}{3} \times$ area of the base $\times$ the height of the pyramid).

Example

Find the volume of the pyramid given.



Solution

Since $V = \frac{1}{3}BH$, you must determine B first.

Step 1: Calculate B, the area of the base.

$$\begin{aligned} B &= \ell w \\ &= 12(9) \\ &= 108 \end{aligned}$$

In this case, the base is a rectangle; thus, $B = \ell w$.

The area of the base of the pyramid is $108\ \text{cm}^2$.

Step 2: Calculate V, the volume of the pyramid.

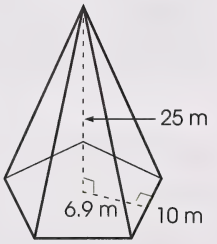
Substitute the area of the base and the height of the pyramid into the volume formula.

$$\begin{aligned} V &= \frac{1}{3}BH \\ &= \frac{1}{3}(108)(15) \\ &= 540 \end{aligned}$$

The volume of the pyramid is $540\ \text{cm}^3$.

Example

Find the volume of the pyramid given.



Solution

Since $V = \frac{1}{3}BH$, you must determine B first.

Step 1: Calculate B, the area of the base.

$$\begin{aligned} B &= \frac{nsa}{2} \\ &= \frac{5(10)(6.9)}{2} \\ &= 172.5 \end{aligned}$$

In this case, the base is a pentagon; therefore, use the formula for the area of a regular polygon.

The area of the base is $172.5\ \text{m}^2$.

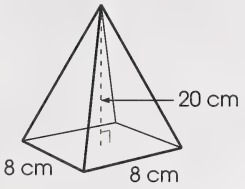
Step 2: Calculate V, the volume of the pyramid.

$$V = \frac{1}{3}BH$$
$$= \frac{1}{3}(172.5)(25)$$
$$= 1437.5$$

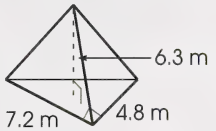
The volume of the pyramid is $1437.5\ \text{m}^3$.

10. Find the volume of each of the following pyramids. Round your answers to the nearest tenth. Refer to the formula sheet at the back of the Appendix if necessary.

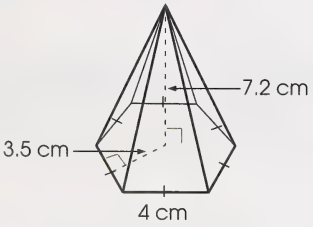
a.



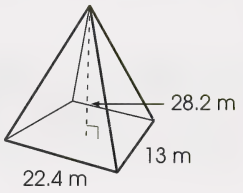
b.



c.



d.



11. The pyramids in Egypt were built in ancient times as monumental tombs for Egyptian kings. The pyramid in the photograph (built around 2700 B.C.) was the tomb of the Egyptian pharoah Khufu (or Cheops). It has a square base, with each side measuring 230 m, and an original height measuring 147 m.

a. Find the volume of stone used to construct this pyramid. Assume that the pyramid is solid stone, with no rooms or passageways.

b. The stone blocks used to build this pyramid were of varying sizes and totalled about 2.3×10^6 (2 300 000) in number. What was the average volume of each stone block? Round your answer to the nearest tenth.

12. The Martins have a tent set up in their backyard that is in the shape of a regular pentagonal pyramid. The area of the floor of the tent is 11.4 m^2, and the height of the tent is 3.2 m. What volume of air is in the tent? Round your answer to the nearest tenth.

13. A cube can be divided into 6 congruent pyramids. Each face of the cube serves as a base for a pyramid. The edges of the cube are 60 cm long. What is the volume of each pyramid? Show your answer in two different ways.

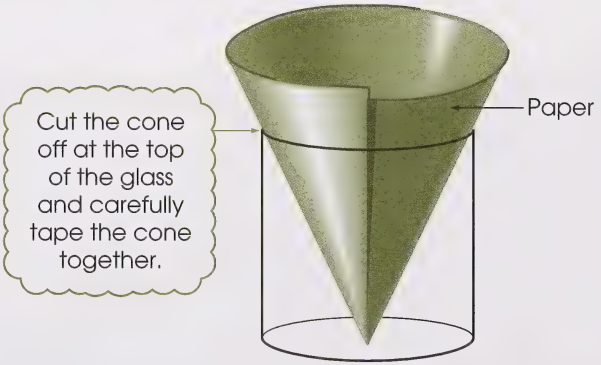
Check your answers by turning to the Appendix, page 121.

Volume of a Cone

You have worked with a formula for the volume of a cylinder. The formula for the volume of a cylinder is the area of the base of the cylinder times the height. There is also a relationship between the volume of a cylinder and the volume of a cone. This relationship will help you to determine volumes like the volume of sand in a cone-shaped pile.

Complete the following steps and answer the questions to discover this relationship.

Step 1: Using an empty glass (or some other cylindrical container), construct a paper cone that just fits inside the glass (as shown in the diagram). Cut the cone appropriately, and tape it together.



Step 2: Fill the cone with rice (or any other small-grained, pourable material), and pour it into the glass.

14. Complete the following to arrive at the formula for the volume of a cone.

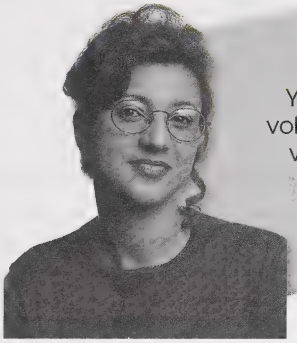
a. Is the radius of the top of the cone the same as the radius of the glass?

b. How many times must you fill the cone and pour it into the glass in order to fill the glass completely?

c. If you know the volume of the glass, how can you find the volume of the cone?

d. Write a formula for the volume of a cone based on your observations. The formula should indicate the volume in terms of the base and height of the cone.

Check your answers by turning to the Appendix, page 123.



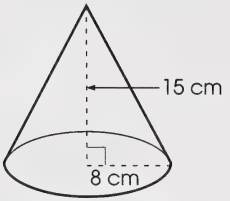
You have discovered that the volume of a cone is one-third the volume of a cylinder with the same base and height.

The volume of any cone is found by using the general formula $V = \frac{1}{3}BH$ $\left(\frac{1}{3} \times \text{area of the base} \times \text{height of the cone}\right)$.

The formula for the volume of a cone can be used to calculate the volume of various cone-shaped figures and objects.

Example

Find the volume of the cone given. Round your answer to the nearest whole number.



Solution

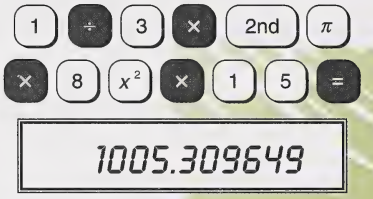
Use the formula for the volume of a cone, and substitute the given dimensions.

$$V = \frac{1}{3}BH$$

$$V = \frac{1}{3}\left(\pi r^2\right)H$$

$$= \frac{1}{3}\pi(8)^2(15)$$

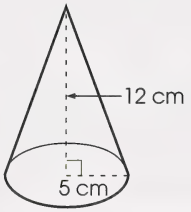
$$\doteq 1005$$



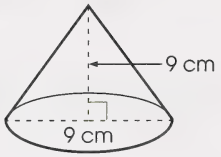
The volume of the cone is approximately 1005 cm^3.

15. Find the volume of each of the following cones. Round your answer to the nearest whole number.

a.



b.



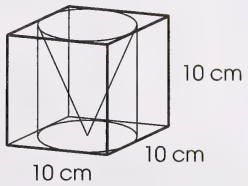
c. a cone with a radius of 6 cm and a height of 14 cm

d. a cone with a diameter of 18 cm and a height of 20 cm

16. A snow cone has a diameter of 8 cm and a height of 12 cm. How much crushed ice can the cone hold if it is filled level to the top? Round your answer to the nearest whole number.

17. The cups used at a water dispenser on a golf course are cone-shaped. If the cups have a radius of 3.5 cm and a height of 7.2 cm, how much water (in mL) can each cup hold? (**Hint:** $1\ \text{cm}^3 = 1\ \text{mL}$) Round your answer to the nearest whole number.

18. The cube shown represents 1 L or 1000 mL.



a. Calculate the capacity of a cylinder that just fits inside the cube. Round your answer to the nearest whole number.

b. Calculate the capacity of a cone that just fits inside the cube. Round your answer to the nearest whole number

c. Expressing as common fractions, approximately what part of the cube is filled by the cylinder and what part of the cube is filled by the cone?

d. Which would fill more of the cube, a cone or a rectangular pyramid that just fits inside? Explain.

Check your answers by turning to the Appendix, page 123.

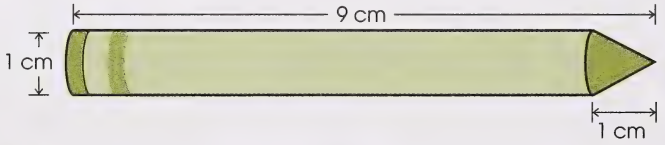
Looking Back

In this activity, you studied volume. You calculated the volumes of right prisms, cylinders, cones, and pyramids. You studied the relationship between the volume of a rectangular prism and the volume of a rectangular pyramid. You also examined the relationship between the volume of a cylinder and the volume of a cone.

In your journal, copy the following formulas for calculating volumes.

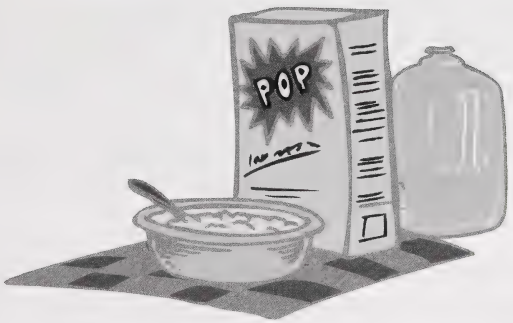
- Volume of right prism = (area of base)(height of right prism)
- Volume of cylinder = (area of base)(height of the cylinder)
- Volume of pyramid = $\frac{1}{3}$(area of base)(height of pyramid)
- Volume of cone = $\frac{1}{3}$(area of base)(height of cone)

19. Calculate the volume of wax needed to make a jumbo crayon. Round your answer to the nearest tenth.



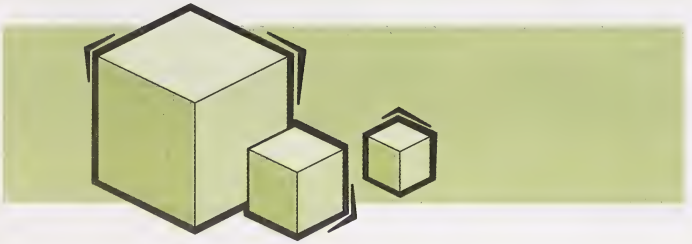
Check your answer by turning to the Appendix, page 124.

Activity 5: Volume and Surface Area in Package Design

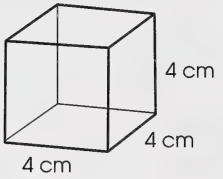


What size of cereal box does your family buy? The size of cereal boxes sold is of great importance to companies that produce and sell cereal. Surface area determines how much material is required to produce the package, while volume indicates how much the box will hold.

In this activity, you will analyse the relationship between dimensions, surface area, S, and volume, V.



You are given a cube similar to the one shown on the right. Use this cube to answer questions 1 to 5.



1. Calculate the surface area and the volume of the cube.

2. Double all the dimensions of the cube.

   a. Calculate the surface area and the volume.

   b. How many times does the surface area increase when the dimensions are doubled?

   c. How many times does the volume increase when the dimensions are doubled?

3. Express the increases in questions 2.b. and 2.c. as powers of 2.

4. Triple all the dimensions of the original cube.

   a. Calculate the surface area and the volume.

   b. How many times does the surface area increase when the dimensions are tripled?

   c. How many times does the volume increase when the dimensions are tripled?

5. Express the increases in questions 4.b. and 4.c. as powers of 3.

6. **a.** Copy and complete the following chart for changes in surface area and volume based on changes in dimensions.

Change in Dimensions	Change in Surface Area	Change in Volume
doubled	$2^2 \times$	$2^3 \times$
tripled		
halved		
n times		

b. Which changes faster for a given change in the dimensions, surface area or volume?

c. Write, in words, the relationship between changes in surface area and changes in dimension, and changes in volume and changes in dimension.

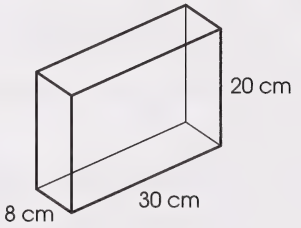
d. Explain why the exponent for change in surface area is 2, while the exponent for change in volume is 3.

Check your answer by turning to the Appendix, page 124.

What effect does changing the dimension of a rectangular prism (similar to a box of cereal) have on its surface area and volume?

Complete questions 7 to 9 to find out.

7. Find the surface area and the volume of this container, which is in the shape of a rectangular prism.



8. Double the dimensions of the container in question 7.

a. Calculate the surface area and the volume.

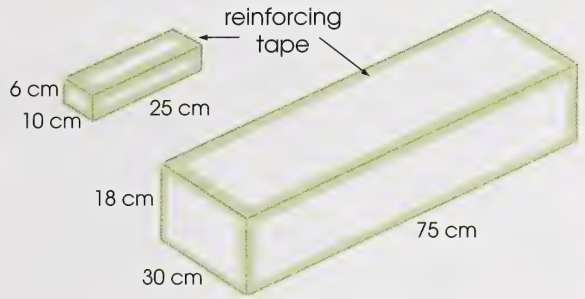
b. Is the relationship between doubling the dimensions and the change in surface area and the change in volume the same as the relationship for a cube?

9. **a.** What effect does doubling the dimensions of a box of cereal have on the material needed to make the box and on the amount of cereal it can hold?

b. What implications does this have for the cereal industry?

c. Why do cereal packages come in a variety of sizes?

10. ABC Manufacturing Inc. has two similar types of packages for one of its products.



a. What is the ratio of the lengths of the sides of the larger box to the lengths of the sides of the smaller box?

b. How much more reinforcing tape is needed for the larger box as compared to the smaller box?

c. What is the relationship in the amount of material needed to build each box?

d. How do the volumes of the two boxes compare?

Check your answer by turning to the Appendix, page 126.

What effect does changing the height or radius of a cylinder (similar to a soup can) have on its surface and volume?

Complete questions 11 to 13 for the answer.

11. A soup can, with a radius of 4 cm and a height of 12 cm, has a surface area of approximately 402 cm^2 and a volume of approximately 603 cm^3.

a. Predict the surface area and the volume of the can if the radius and height are doubled.

b. Test your prediction in question 14.a. Round your answers to the nearest whole number. **Remember:** The surface area includes both the top and bottom of the can, while volume is the area of one end times the height.

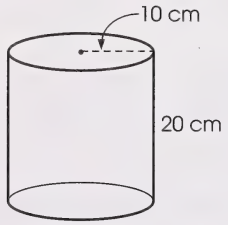
12. a. Copy and complete the following chart to find the surface area and the volume for various heights of cans with a radius of 4 cm. Round your answers to the nearest whole number.

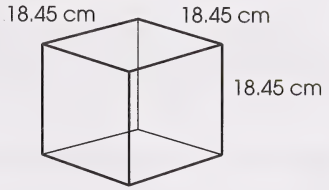
Height, h (cm)	Surface Area, S (cm^2)	Volume, V (cm^3)
4	201	201
5		
6		
7		
8		
12		

b. Draw the graphs of surface area related to height, and then volume related to height. Label each graph. How do the graphs compare?

c. Join each set of points with a straight line and extend each line back to the y-axis. What is the surface area of a can with a height of 0 cm? Why? What is the volume of a can with a height of 0 cm? Why?

13. The following two containers have the same volume.





a. Calculate the surface area of the cylindrical container. Round your answer to the nearest whole number.

b. Calculate the surface area of the cube. Round your answer to the nearest whole number.

c. How do the surface areas compare?

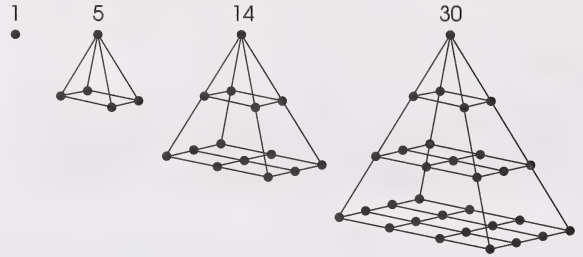
d. What does this tell you about surface area versus volume for these containers with different shapes? How does this affect the packaging industry?

Check your answers by turning to the Appendix, page 128.

Now Try This

Solve the following challenge problem on pyramidal numbers.

14. You are familiar with even numbers $(2, 4, 6, 8, \ldots)$, odd numbers $(1, 3, 5, 7, \ldots)$, and square numbers $(1, 4, 9, 16, \ldots)$; however, have you heard of pyramidal numbers? Pyramidal numbers are illustrated in the following diagram.



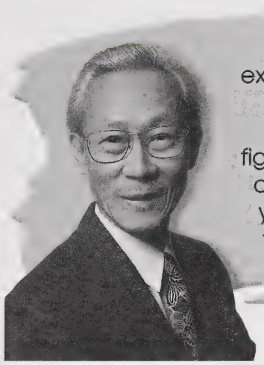
a. What would the next pyramidal number be?

b. What kind of numbers make up each row?

c. How would you find the tenth pyramidal number?

Check your answers by turning to the Appendix, page 131.

Looking Back



In this activity, you explored the effects of changing the dimensions of solid figures on their surface area and volume. In your journal, do the following question.

15. You have been hired by a local movie theatre to design an open container with the following specifications:

- will be a square base
- will hold 4000 cm^3 of popcorn
- will require the least amount of material to make

Hint: A popcorn container used in a movie theatre does not require a top.

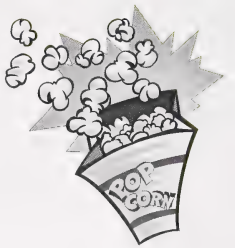
a. Determine the container with a square base that holds approximately 4000 cm^3 of popcorn and has the least surface area. Copy and complete a chart similar to the following using trial and error. Round your answer to the nearest hundredth where necessary.

Volume (cm^3)	Length (cm)	Width (cm)	Height (cm)	Surface Area (cm^2)
4000	10	10		

b. What shape of container might be more efficient? Experiment with various values for radius and height to determine the most efficient shape, with the most efficient dimensions.

c. What factors other than minimum surface area might influence the size and shape of a container?

Check your answers by turning to the Appendix, page 131.



Conclusion

In this section, you investigated how algebra is related to everyday calculations. You discovered the meaning of a variable and used variables to write variable expressions. You then evaluated variable expressions for various values of the variable. Finally, you used algebra to describe the volume and surface areas of a variety of shapes.

You discovered that almost everyone uses algebra. People use algebra to calculate the perimeter of a room when they are buying new baseboards and calculate the area of a living-room floor when they are installing new carpets. Business people use algebra when they work with spreadsheets. Economists use algebra to show economic trends and make predictions. Police officers use algebra to calculate the braking distance at various speeds. These are just a few of many examples of how algebra is used in the everyday world.

Assignment

Turn to Assignment Booklet 3A and complete the assignment for Section 1.

SECTION 2

Solving Equations

Riding a bicycle is an activity that many people of all ages enjoy. Experienced riders usually can keep their balance; once the bicycle is in motion, balance is maintained by steering skillfully. Maintaining balance is especially important during cycling competitions.

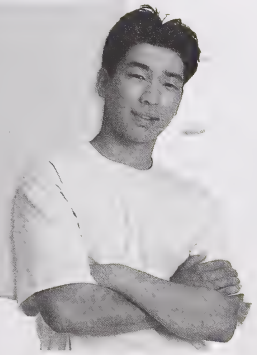
Maintaining balance is also important when solving mathematical equations. The amount on the left side of the equation must equal the amount on the right side of the equation. For this reason, a balance scale is often used to model equations. The scale is a reminder that whenever an operation is performed on one side of an equation, the same operation must be performed on the other side of the equation.

In this section, you will model, solve, and verify equations; use equations to solve problems; and evaluate formulas using equation-solving skills.

Activity 1: Solving Equations with a Variable on One Side

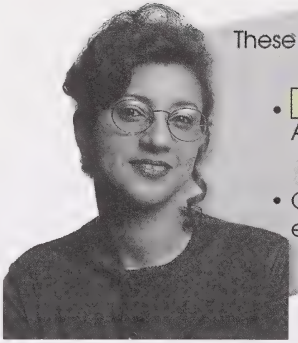
Both of these sentences are true:

- Edmonton is the capital city of Alberta.
- Four more than five is nine.



Both of these sentences are false:

- Red Deer is the largest city in Alberta.
- Seven less than ten is two.



These sentences may be true or false:

- ☐ is the premier of Alberta.
- One plus ☐ gives eight.

1. Write each of the following sentences using algebraic symbols. Let each ☐ be represented by x.

a. Five plus ☐ results in thirteen.

b. Four more than five is ☐.

c. Three less than eight gives the same result as ☐ plus four.

d. One more than four is the same as two less than ☐.

2. Replace the variable in each equation in question 1 with a number to make the statements true.

Check your answers by turning to the Appendix, page 132.

Each of the equations in question 1 is a conditional equation.

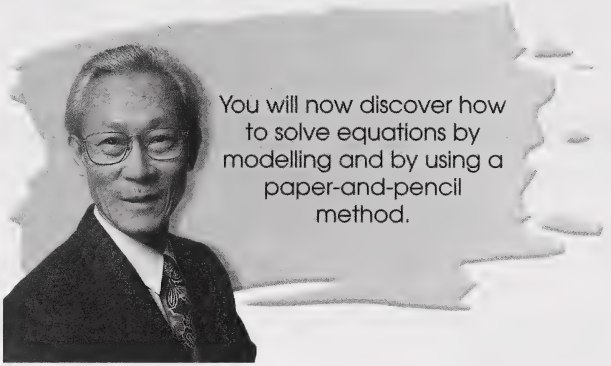
A conditional equation is an equation that is only true for certain values of the variable.

The value of the variable that makes an equation true is called a solution. Finding the value of the variable that makes the equation true is called solving an equation.

You probably solved each of the equations in question 1 by guessing, checking, and revising your guesses until you found a value that made the equation true. In this activity, you will discover a more systematic method of solving equations.

To help you visualize the process of solving equations, you will use positive and negative counters and cylinders as well as an equation scale model. The equation scale (a balance scale) will help you remember that whenever an operation is performed on one side of an equation, the same operation is performed on the other side of the equation.

At the back of the Appendix, find the pages with positive and negative counters, with positive and negative cylinders, and with the equation scale. Photocopy these pages or glue them to heavier paper or boxboard (cereal boxes) before cutting them out. This will make these learning aids easier to handle.



Study the following examples. In each example, the modelling method of solving the equation is shown first. The paper-and-pencil method is shown second. Notice the similarity between the two methods.

Use your cut-out learning aids to follow along.

Example

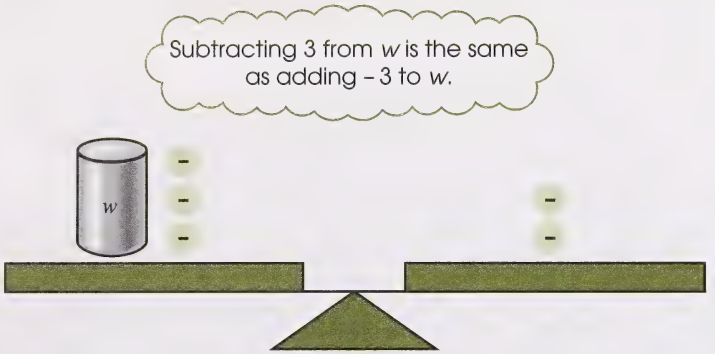
Solve the equation $w - 3 = -2$ and verify the solution.

Solution

Method 1: Using Models

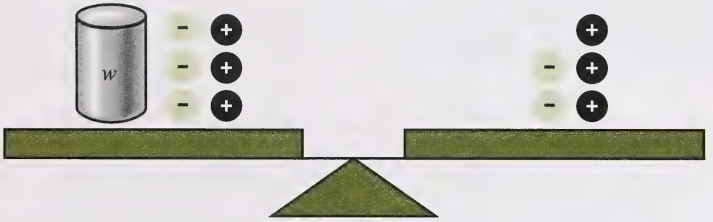
To solve the equation $w - 3 = -2$, first model it.

Put $w-3$ on one side of the scale. This equals the amount on the other side of the scale, -2.



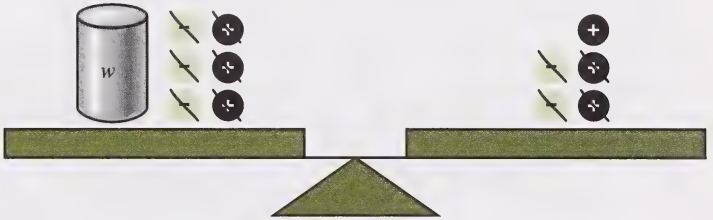
Ask yourself, "How can I isolate w?" "How can I get w by itself?"

You can isolate w by undoing the addition of -3. That is, add $+3$ to **each** side of the equation.

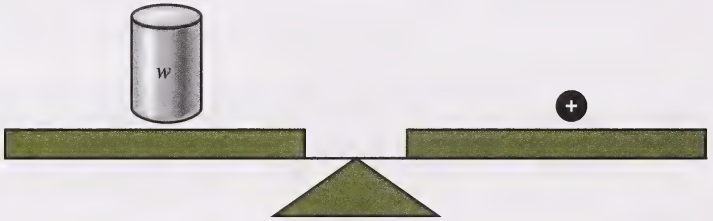


Adding $+3$ to each side of the equation maintains the balance.

Simplify **each** side by removing any zero pairs. **Note:** A zero pair consists of a negative counter and a positive counter.



This is the result; w has been isolated.



$\therefore\ w=1$

Method 2: Using Paper and Pencil

To solve the equation $w-3=-2$, ask yourself, "How can I isolate w?"

You can isolate w by undoing the subtraction of 3. That is, add 3 to **each** side of the equation.

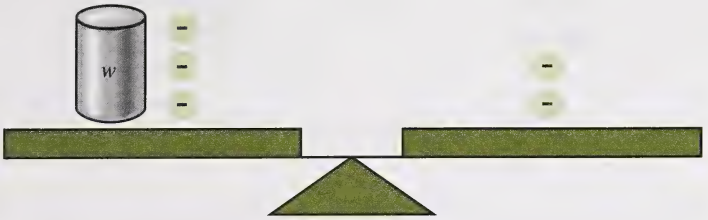
$$w-3+3=-2+3$$
$$w=1$$

$$\begin{array}{rcl} w-3 & = & -2 \\ +3 \quad +3 & & \\ \hline w & = & 1 \end{array}$$

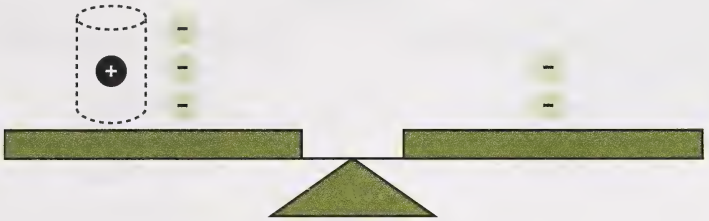
Check

Method 1: Using Models

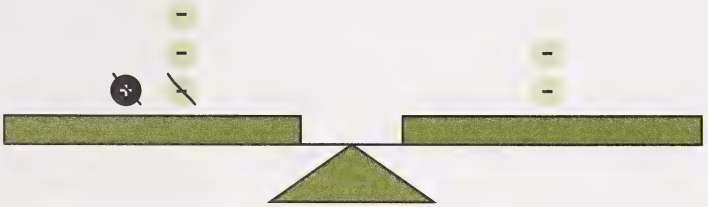
To verify that $w = 1$, first model the equation $w - 3 = -2$.



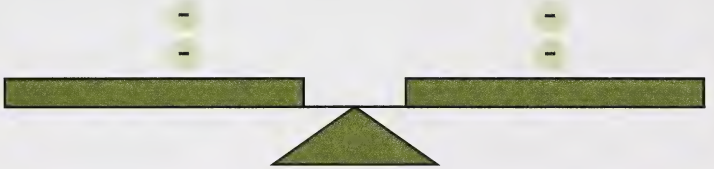
Replace w with one positive counter.



Simplify the left-hand side by removing any zero pair.



The left side equals the right side.

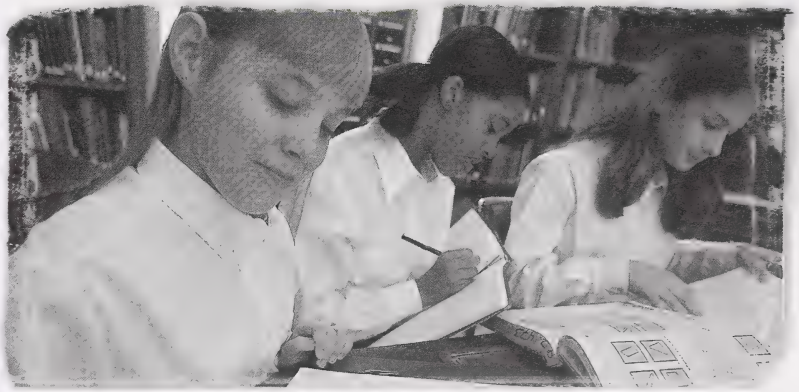


Method 2: Using Paper and Pencil

Substitute $w = 1$ into the equation. Does the left side equal the right side?

LS	RS
$w - 3$	-2
$= 1 - 3$	
$= -2$	

LS = RS



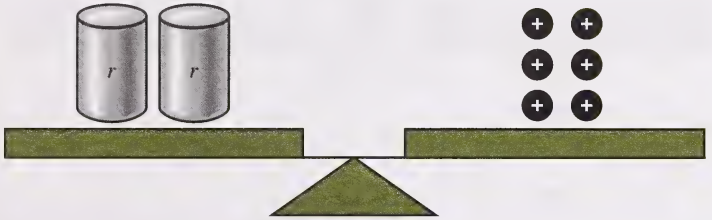
Example

Solve the equation $2r = 6$, and verify the solution.

Solution

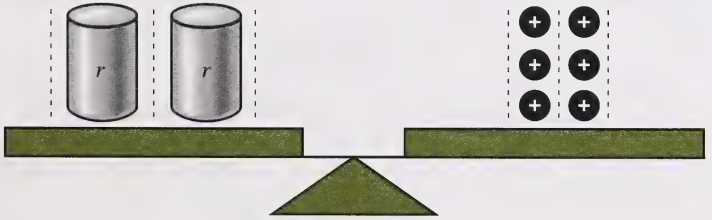
Method 1: Using Models

To solve the equation $2r = 6$, first model it.

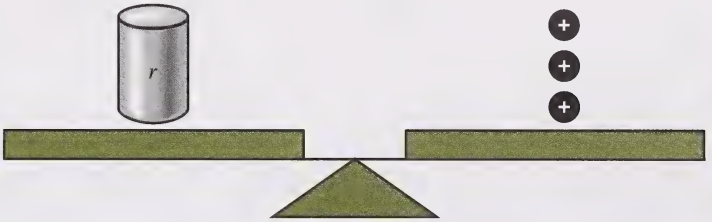


Notice that the cylinders are alone on one side of the equation; however, there are two cylinders, not one.

To solve for r, first arrange the learning aids on **each** side of the equation into two equal groups.



Keep **one** group from **each** side of the scale. This is the same as dividing each side by 2.



$\therefore\ r = +3$

Method 2: Using Paper and Pencil

To solve the equation $2r = 6$, ask yourself, "How can I get $1r$ on the left side of the equation?"

You can get $1r$ on the left side of the equation by dividing **each** side of the equation by 2.

$$2r = 6$$
$$\frac{2r}{2} = \frac{6}{2}$$
$$r = 3$$

Alternatively, you can get $1r$ on the left side of the equation by multiplying each side of the equation by $\frac{1}{2}$.

$$2r = 6$$
$$\frac{1}{\cancel{2}_{1}}\left(\cancel{2}^{1}r\right) = \frac{1}{\cancel{2}_{1}}\left(\cancel{6}^{3}\right)$$
$$r = 3$$

Check

Substitute $r = 3$ into the equation. Does the left side equal the right side?

LS	RS
$2r$	6
$= 2(3)$	
$= 6$	

LS = RS

Example

Solve the equation $\frac{x}{3} = 5$ and verify the solution.

Solution

To solve the equation $\frac{x}{3} = 5$, ask yourself, "How can I get $1x$ on the left side of the equation?"

You know that $\frac{x}{3}$ means $\frac{1}{3} \times x$ and that the inverse operation is multiplying by $\frac{3}{1}$ or 3.

Therefore, you can get $1x$ on the left side of the equation by multiplying **each** side of the equation by 3, since the multiplicative inverse of $\frac{1}{3}$ is 3.

$$\frac{x}{3} = 5$$
$$\cancel{3}^{1}\left(\frac{x}{\cancel{3}_{1}}\right) = 3(5)$$
$$x = 15$$

Check

Substitute $x = 15$ into the equation. Does the left side equal the right side?

LS	RS
$\frac{x}{3}$	5
$= \frac{15}{3}$	
$= 5$	

LS = RS

3. In each of the following equations, what operation should be performed to solve the equation?

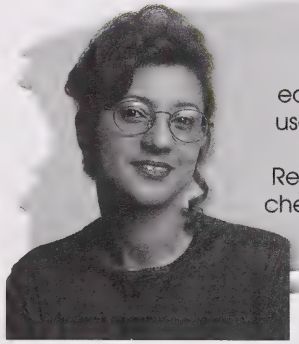
a. $x+2=7$ **b.** $t-5=1$ **c.** $m+3=-1$

d. $y-2=-8$ **e.** $3b=-6$ **f.** $-2s=-12$

g. $\frac{n}{4}=3$ **h.** $\frac{m}{2}=-6$

4. Solve each equation in question 3, and verify.

Check your answers by turning to the Appendix, page 132.



Conditional equations can be used when solving problems. Remember, always check your answers.

Example

Fatima received an allowance of $25. She deposited this money into her savings account. If Fatima's new balance was $81, how much was her previous balance?

Solution

Step 1: Identify the unknown number or quantity and choose a variable to represent it.

Let Fatima's previous balance be p.

Step 2: Write an equation to describe the situation.

$$p+25=81$$

Step 3: Solve the equation and answer the question.

$$p+25-25=81-25$$
$$p=56$$

$$\begin{array}{rl} p+25= & 81 \\ -25 & -25 \\ \hline p = & 56 \end{array}$$

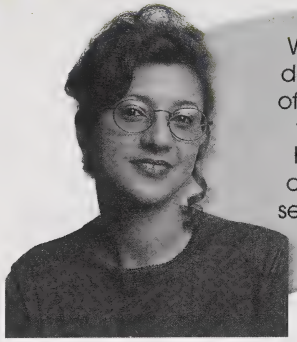
Fatima's previous balance was $56.

Check

Determine if the answer is reasonable.

Ask yourself, "Would Fatima have a total of $81 if she added $25 to $56?"

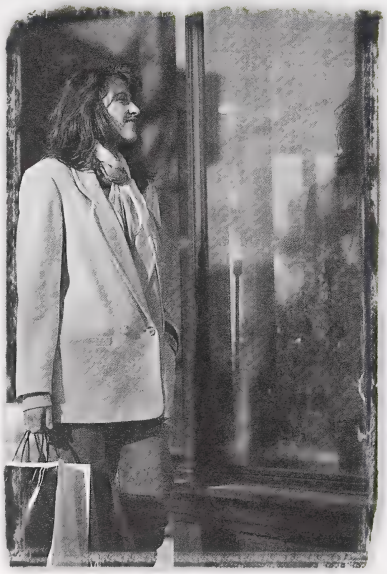
Yes, the answer is reasonable.



When checking your solution, don't merely verify the solution of the equation; you may have written the wrong equation. Instead, reread the problem and substitute your solution to see if the answer is reasonable.

Example

At a local boutique, dresses are on sale for $\frac{2}{3}$ of their regular price. The sale price of a dress is \$80. What is the regular price of the dress?



Solution

Step 1: Write an equation. Let r be the regular price.

$$\frac{2r}{3} = 80$$

Step 2: Multiply each side of the equation by $\frac{3}{2}$, since $\frac{2}{3} \times \frac{3}{2} = 1$.

$$\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}}}\left(\frac{\overset{1}{\cancel{2}}r}{\underset{1}{\cancel{3}}}\right) = \frac{3}{\underset{1}{\cancel{2}}}\left(\overset{40}{\cancel{80}}\right)$$

$$r = 120$$

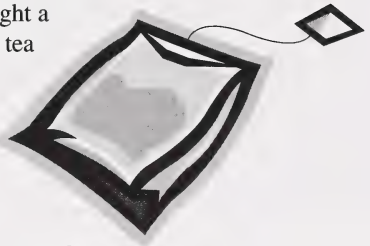
The regular price of the dress was \$120.

Check

Is $\frac{2}{3}$ of \$120 equal to \$80?

Yes, the answer is reasonable. The regular price of the dress was \$120.

For questions 5 to 11, develop equations and solve for the unknown. Then determine if each answer is reasonable.



5. Joe had only 5 tea bags left. So, he bought a new box of tea bags. If he now has 125 tea bags, how many tea bags were in the box Joe purchased?

6. Chandia used an average of 6 sheets of loose-leaf paper per day. At the end of the term, she had used 654 sheets of paper. How many days were there in the term?

7. Jeannette bought two bus tickets for \$23. One ticket was a child's ticket and the other an adult's ticket.

If the child's ticket cost \$5, how much did the adult ticket cost?

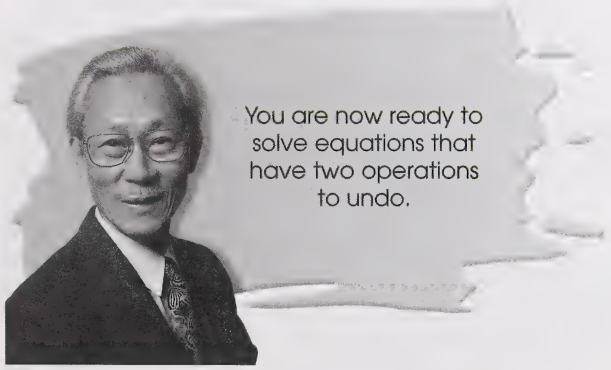
8. The width of a rectangular card is 8 cm less than the length. If the width is 12 cm, what is the length?

9. Maria had a length of fabric to make banners. She cut the fabric into six equal pieces. If each piece was 2.75 m long, what was the original length of the fabric?

10. At matinée performances, the cost of admission to a movie is $\frac{3}{4}$ of the regular price. If Lloyd and Heather each pay \$6 for admission to a matinée performance, what is the regular price?

11. Sweaters are on sale for $\frac{1}{3}$ off the regular price. If the sale price is \$12, what is the regular price? **Hint:** $\frac{1}{3}$ off is the same as $\frac{2}{3}$ of the regular price.

Check your answers by turning to the Appendix, page 138.



Study the following example. The modelling method is given first, and the paper-and-pencil method is shown second. Notice the similarity between the two methods.

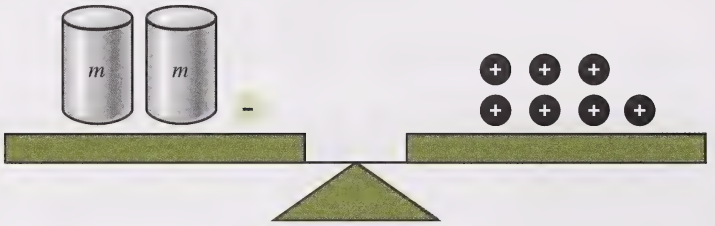
Example

Solve the equation $2m - 1 = 7$, and verify the solution.

Solution

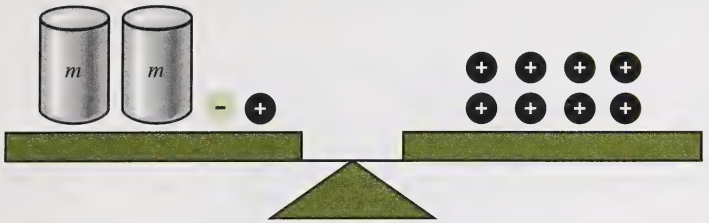
Method 1: Using Models

Model the equation.

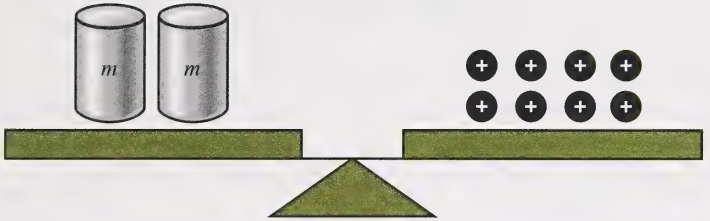


Ask yourself, "How can I isolate the cylinders?"

You can isolate the cylinders by undoing the addition of one negative counter. That is, add one positive counter to **each** side of the scale.

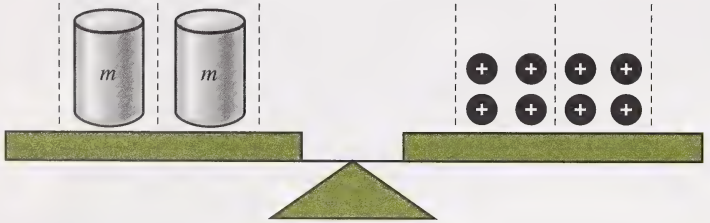


Simplify the equation by removing any zero pairs. This is the result.

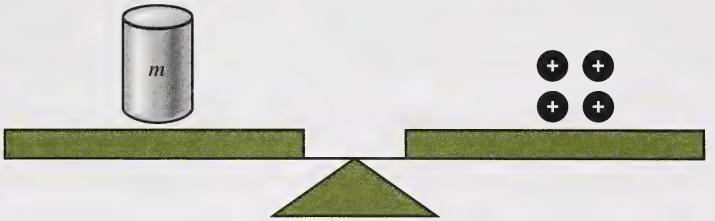


Now, solve for m.

Arrange the learning aids on **each** side of the equation into two equal groups.



Keep **one** of the groups from **each** side of the scale.



$\therefore\ m = 4$

Method 2: Using Paper and Pencil

Step 1: Isolate the variable.

$$2m - 1 = 7$$
$$2m - 1 + 1 = 7 + 1$$
$$2m = 8$$

Add +1 to each side.

Step 2: Solve for m.

$$\frac{2m}{2} = \frac{8}{2}$$
$$m = 4$$

Divide each side by 2.

or

$$\frac{1}{\cancel{2}_1}\left(\frac{1}{\cancel{2}}m\right) = \frac{1}{\cancel{2}_1}\left(\frac{4}{\cancel{8}}\right)$$
$$m = 4$$

Multiply each side by one-half.

Check

Substitute $m = 4$ into the equation. Does the left side equal the right side?

LS	RS
$2m-1$	7
$=2(4)-1$	
$=8-1$	
$=7$	

LS = RS

12. Solve each of the following equations using the paper-and-pencil method. Verify each solution.

a. $3a-2=7$

b. $2b+1=-9$

c. $\frac{c}{3}-4=5$

d. $\frac{d}{2}+1=5$

Check your answers by turning to the Appendix, page 140.

Looking Back

In this activity, you solved equations by modelling and by using paper and pencil. You verified your equations and discovered that you didn't necessarily need to check your answer in the Appendix because you could determine whether your answer was correct or incorrect yourself.

As the equations get more complicated, it is important to read the equations to yourself to determine which mathematical operations are being performed and, therefore, which operations will have to be done to "undo" these operations.

For example, if you see the equation $5x = 30$, you should read it as "5 times x equals 30." *Times* means multiply; so, you need to divide to undo the operation. The thought process should be, "I need to divide both sides of the equation by 5."

$$\frac{5x}{5} = \frac{30}{5}$$
$$x = 6$$

Check your answer. Does 5 times 6 equal 30? Yes.

13. In your journal, write the equation $\frac{x}{3}+2=6$ in words.

a. To isolate the variable, which two operations will have to be done?

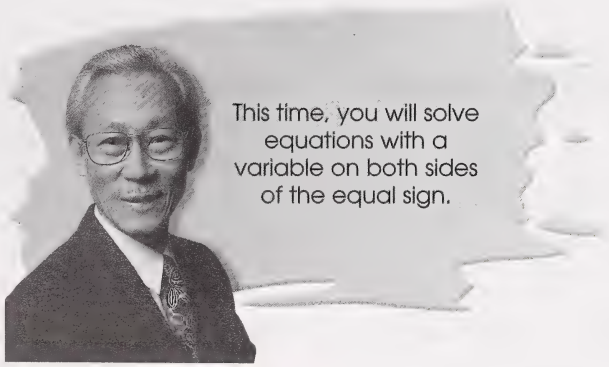
b. Which operation will have to be done first? Why?

Check your answers by turning to the Appendix, page 141.

Activity 2: Solving Equations with a Variable on Each Side

Models are often used in the everyday world to help people visualize concepts. For example, a globe may be used to give people a sense of where places are located.

In this activity, you will continue to use learning aids to help you visualize the process of solving equations.



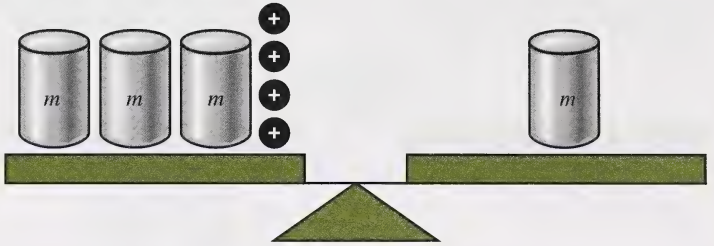
Example

Solve the equation $3m + 4 = m$. Verify your solution.

Solution

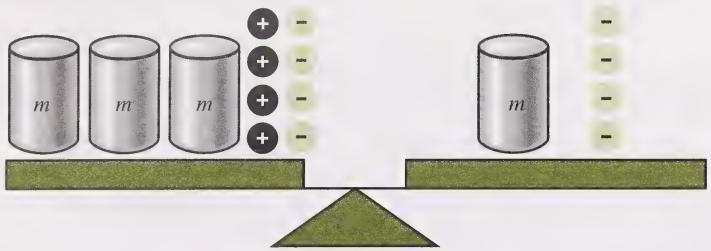
Method 1: Using Models

Model the equation.

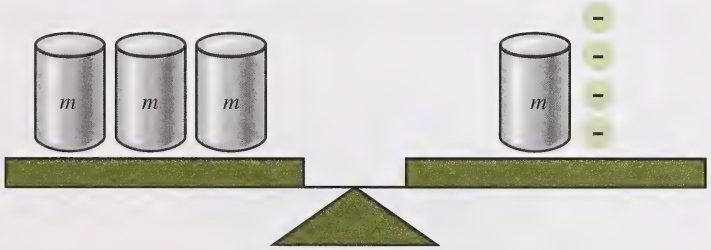


The equation $3m + 4 = m$ has variables on both sides of the equals sign. To solve this equation, you will have to separate the variables and the constants so that the variables are on one side of the equation and the constants are on the other side.

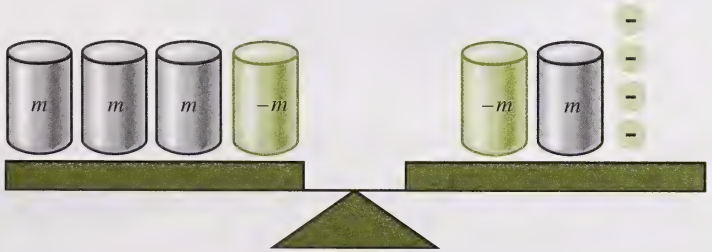
To isolate $3m$ on the left side, add -4 to both sides, since -4 is the opposite or additive inverse of $+4$.



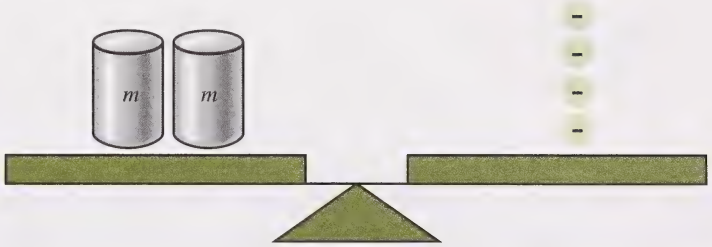
Simplify by removing any zero pairs. This is the result.



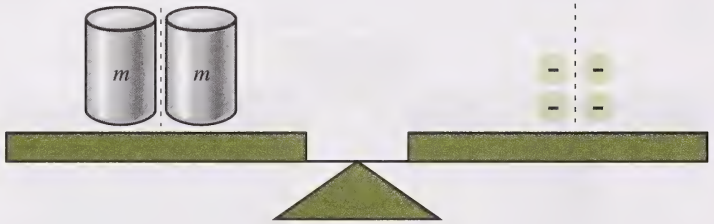
To isolate -4 on the right side, add $-m$ to both sides, since $-m$ is the inverse of m.



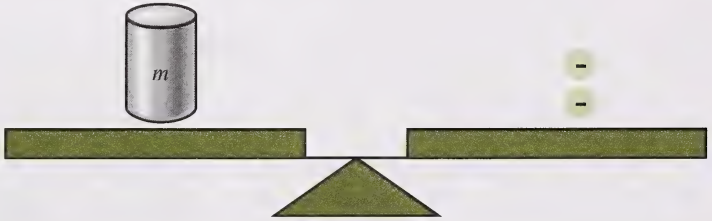
Simplify by removing the zero pairs. This is the result.



To isolate m, divide each side into two groups.



To solve the equation, keep **one** of the groups from **each** side of the scale. This is the result.



$\therefore\ m = -2$

Method 2: Using Paper and Pencil

Write the equation.

$$3m + 4 = m$$

To isolate $3m$ on the left side of the equation, subtract 4 from both sides; then simplify.

$$3m + 4 - 4 = m - 4$$
$$3m = m - 4$$

To isolate -4 on the right side of the equation, subtract m from both sides; then simplify.

$$3m - m = m - 4 - m$$
$$2m = -4$$

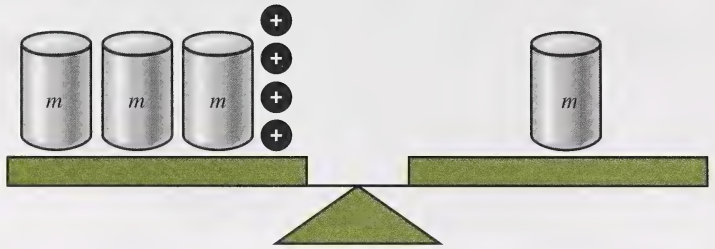
To isolate m, divide both sides by 2.

$$\frac{2m}{2} = \frac{-4}{2}$$
$$m = -2$$

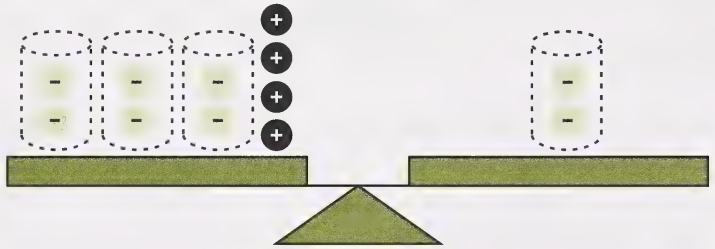
Check

Method 1: Using Models

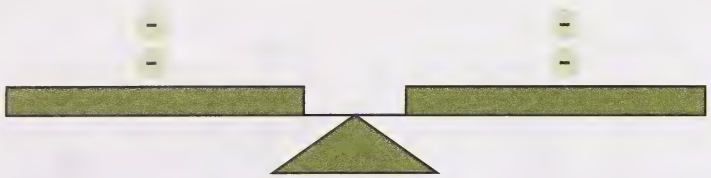
Model the equation.



Replace each m with -2 (two negative counters).



Simplify by removing the zero pairs. The result is this.



The value of each side of the equation is -2. The scale is balanced.

Therefore, the solution is $m = -2$.

Method 2: Using Paper and Pencil

LS	RS
$3m+4$	m
$=3(-2)+4$	$=-2$
$=-6+4$	
$=-2$	

LS = RS

Therefore, the solution is $m = -2$.

1. The example you just examined models the solution of $3m+4=m$. There are variables on both sides of the equation.

a. Why was -4 added to both sides of the equation?

b. Why was $-m$ added to both sides of the equation?

c. Why were both sides of the equation divided by 2?

2. Solve the following equations using paper and pencil.

a. $5n-8=3n$

b. $10-d=-6d$

c. $4n+12=-2n$

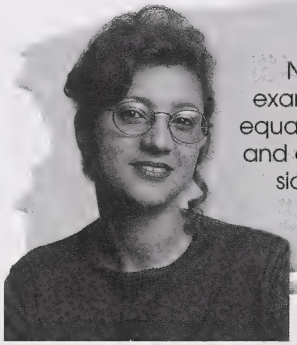
d. $3n+48=5n$

e. $3.2a-14=1.2a$

f. $1.6y=2.8y-6$

Check your answers by turning to the Appendix, page 141.

Sometimes there is a variable and a constant on each side of the equation. The same procedure is used; it will just take more steps.



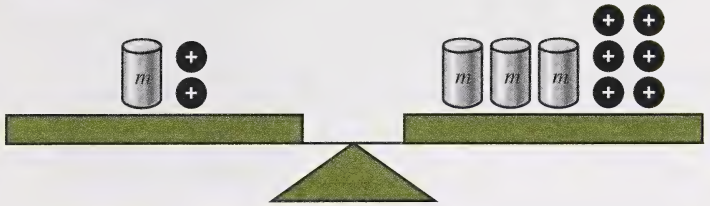
Now, follow an example in which an equation with variables and constants on both sides is solved by modelling.

Example

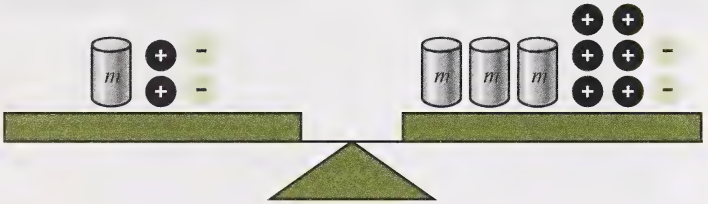
Solve the equation $m+2=3m+6$ by modelling. Verify the solution.

Solution

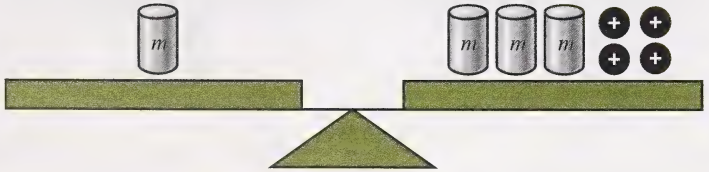
Model the equation.



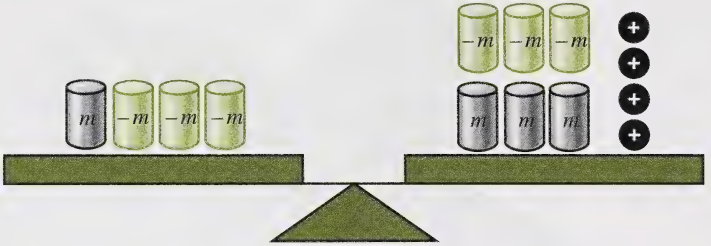
To isolate m on the left side of the equation, add -2 to both sides.



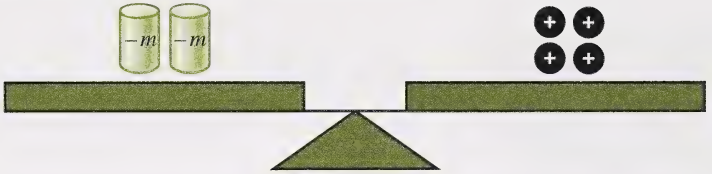
Simplify by removing the zero pairs. The result is this.



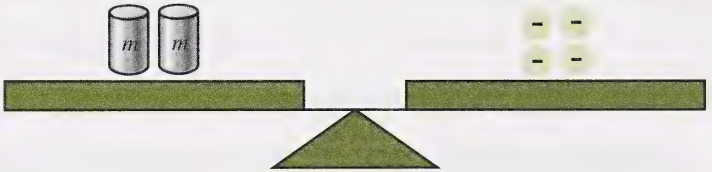
To isolate $+4$ on the right side of the equation, add $-3m$ to both sides.



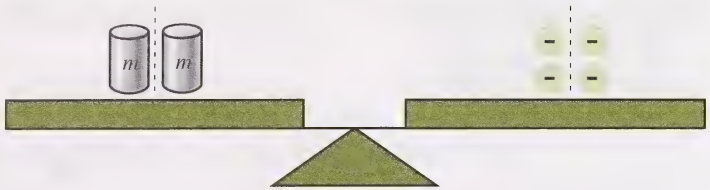
Simplify by removing any zero pairs. The result is this.



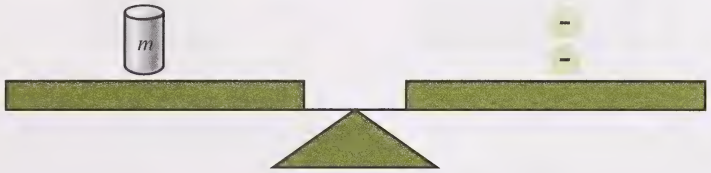
Replace each side with its inverse.



Divide each side into two groups.



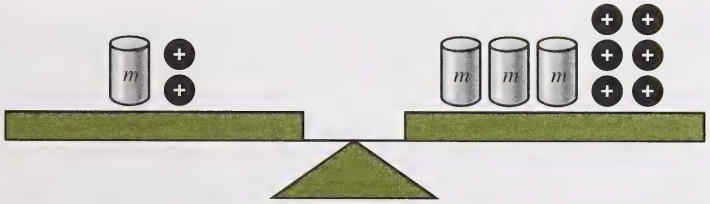
To solve the equation, keep **one** group from **each** side of the scale. This is the result.



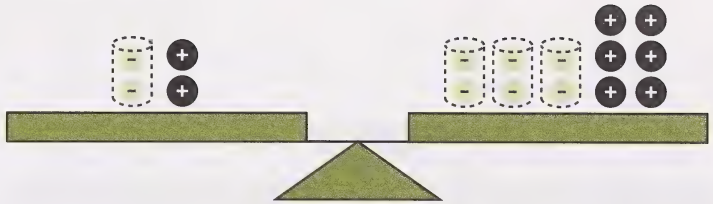
$\therefore\ m = -2$

Check

Model the equation.



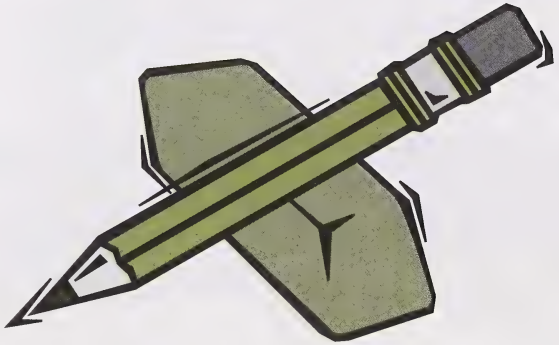
Replace m with -2.

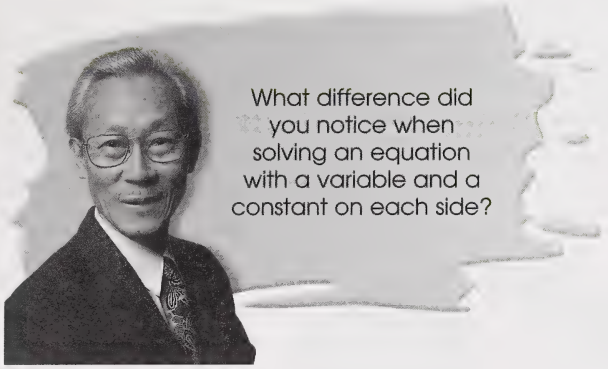


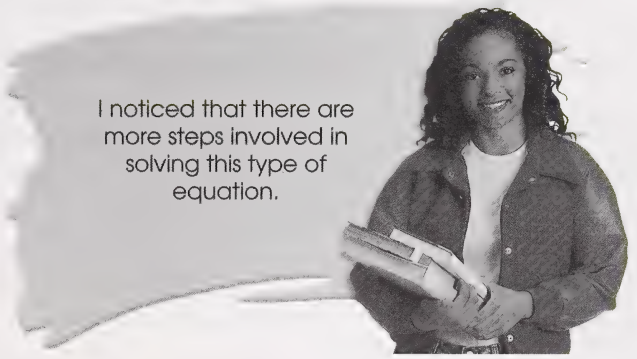
Simplify by removing the zero pairs. The result is this.

The value of each side of the equation is zero. The scale is balanced.

Therefore, the solution is $m = -2$.







Now, you will see how to solve the preceding example using the paper-and-pencil method.

Example

Solve the equation $m+2=3m+6$ using the paper-and-pencil method; then verify the solution.

Solution

Write the equation.

$$m+2=3m+6$$

To isolate m on the left side, subtract 2 from both sides.

$$m+2-2=3m+6-2$$
$$m=3m+4$$

To isolate 4 on the right side, subtract $3m$ from both sides.

$$m-3m=3m+4-3m$$
$$-2m=4$$

To solve for m, divide both sides by -2.

$$\frac{-2m}{-2}=\frac{4}{-2}$$
$$m=-2$$

Check

LS	RS
$m+2$	$3m+6$
$=-2+2$	$=3(-2)+6$
$=0$	$=0$
LS =	RS

Therefore, the solution is $m=-2$.

3. Solve the following equations using paper and pencil.

a. $3x+9=2x-5$

b. $4-8a=2a-66$

c. $3+2a=-5a-11$

d. $8n-2=6n+8$

4. Explain how you would model the solution of the equation $3x-2=x$.

Check your answers by turning to the Appendix, page 142.

Example

Solve the equation $0.4q+8.3=0.2q-0.7$; then verify the solution.

Solution

$$0.4q+8.3=0.2q-0.7$$
$$0.4q+8.3-8.3=0.2q-0.7-8.3$$
$$0.4q=0.2q-9.0$$
$$0.4q-0.2q=0.2q-9.0-0.2q$$
$$0.2q=-9.0$$
$$\frac{0.2q}{0.2}=\frac{-9.0}{0.2}$$
$$q=-45$$

Check

LS	RS
$0.4q+8.3$	$0.2q-0.7$
$=0.4(-45)+8.3$	$=0.2(-45)-0.7$
$=-18+8.3$	$=-9-0.7$
$=-9.7$	$=-9.7$
LS =	RS

Therefore, the solution is $q=-45$.

5. Solve the following equations using paper and pencil.

   a. $2.6c - 2.5 = 1.2c + 5.2$

   b. $1.1 + 3.2s = 5s - 11.86$

6. Anton works at a local fast-food outlet. His father, Mr. Mironov, works as a licensed mechanic, where he makes three times more per hour than his son. When Anton was lamenting about the discrepancy, his father told him not to worry; as soon as Anton's wage was increased by $11.60 per hour, they would be "equals." Find their hourly wage.

7. The equations $P_1 = 5.90 + 0.90t$ and $P_2 = 7.25 + 0.45t$ represent the cost of a small pizza from two different pizza shops, where P is the price (in dollars) of a pizza and t is the number of toppings. Use the equations to determine the number of toppings for which the cost is equal.

   Hint: If the cost of both pizzas are equal, the equations representing the cost must be equal.

8. Write a problem to represent the equation $n + 17.40 = 4n$, where n is Lisa's hourly wage and $4n$ is her supervisor's hourly wage.

Check your answers by turning to the Appendix, page 143.

Equations Involving Brackets

Sometimes you will see equations that look quite complicated because they have variables and constants on each side of the equation and they also contain brackets. These equations are not that different or complicated. They are just a combination of the types of equations you have already solved.

Work through the following example.

Example

Solve the equation $3(-2x + 4) = -4(x - 6)$; then verify the solution.

Solution

$$3(-2x+4) = -4(x-6)$$
$$-6x + 12 = -4x + 24$$
$$-6x + 12 - 12 = -4x + 24 - 12$$
$$-6x = -4x + 12$$
$$-6x + 4x = -4x + 12 + 4x$$
$$-2x = 12$$
$$\frac{-2x}{-2} = \frac{12}{-2}$$
$$x = -6$$

Check

LS	RS
$3(-2x+4)$	$-4(x-6)$
$=3[-2(-6)+4]$	$=-4(-6-6)$
$=3(12+4)$	$=-4(-12)$
$=3(16)$	$=48$
$=48$	

LS = RS

Therefore, the solution is $x=-6$.

9. Solve the following equations using paper and pencil. Verify your answers.

a. $5(3x-8)=2(2x+13)$

b. $4(a+6)=8(a+4)$

c. $-4(2x-3)=2(-x+3)$

d. $0.6(2h+6)=1.4(h+2)$

Check your answers by turning to the Appendix, page 144.

Using Tables as a Problem-Solving Strategy

Even though you are using an equation to represent the situation of a problem, sometimes it helps to draw a table or diagram to organize your thoughts and plans when attempting to solve the problem. A table can be used to show relationships, thus helping in setting up the equation.

Example

Mrs. Sanchez is 9 times older than her daugher Carmen. In 3 years, Mrs. Sanchez will only be 5 times older than her daughter. Find their present ages.

Solution

Use a table to organize the information.

Person	Present Age	Age in Three Years
Mrs. Sanchez	$9n$	$9n+3$
Carmen	n	$n+3$

Add three years to their present ages.

In three years, Mrs. Sanchez will be five times Carmen's age. Use this information to write an equation. Then solve the equation.

$$\begin{aligned} 9n+3 &= 5(n+3) \\ 9n+3 &= 5n+15 \\ 9n+3-3 &= 5n+15-3 \\ 9n &= 5n+12 \\ 9n-5n &= 5n+12-5n \\ 4n &= 12 \\ \frac{4n}{4} &= \frac{12}{4} \\ n &= 3 \end{aligned}$$

Now, find Mrs. Sanchez's present age.

$$\begin{aligned} 9n &= 9(3) \\ &= 27 \end{aligned}$$

Check

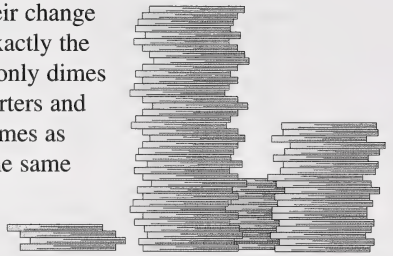
Reread the problem, and be sure your answer is true.

In three years, Mrs. Sanchez will be 30 years old and Carmen will be 6 years old. Is 30 equal to 5 times 6? Yes.

Therefore, Carmen is presently 3 years old and Mrs. Sanchez is presently 27 years old.

Example

Ahmed and Danny were counting their change one day. They discovered they had exactly the same amount of money. Ahmed had only dimes and nickels, and Danny had only quarters and pennies. Ahmed had 2 more than 3 times as many nickels as dimes. Danny had the same number of quarters as Ahmed had dimes and 3 more pennies than he had quarters. How much money did each boy have?



Solution

Use a table to organize the information.

Person	Type of Coin	Number of Coins	Value of One Coin (¢)	Total Value
Ahmed	dime	d	10	$10d$
	nickel	$3d+2$	5	$5(3d+2)$
Danny	quarter	d	25	$25d$
	penny	$d+3$	1	$d+3$

Since the boys have the same amount of money, you can write the following equation:

$$10d + 5(3d + 2) = 25d + (d + 3)$$

Solve the equation to determine the answer to the problem.

$$\begin{aligned}
10d + 15d + 10 &= 25d + d + 3 \\
25d + 10 &= 26d + 3 \\
25d + 10 - 10 &= 26d + 3 - 10 \\
25d &= 26d - 7 \\
25d - 26d &= 26d - 7 - 26d \\
-d &= -7 \\
d &= 7
\end{aligned}$$

Determine the amount of money each boy has. Since they both have the same amount of money, you only need to determine the amount that one of them has. Determine the amount Ahmed has.

$$\begin{aligned}
10d + 5(3d + 2) &= 10(7) + 5\left[3(7) + 2\right] \\
&= 70 + 5(21 + 2) \\
&= 70 + 5(23) \\
&= 70 + 115 \\
&= 185
\end{aligned}$$

Each boy has 185¢ or $1.85 in his pocket.

Check

Ahmed had 7 dimes. Since Ahmed had two more than three times as many nickels, he had 23 nickels. Therefore, Ahmed had $0.70 in dimes and $1.15 in nickels in his pocket. This totals $1.85.

Danny had the same number of quarters as Ahmed had dimes; so, Danny had 7 quarters. Danny had three more pennies than quarters; so, he had 10 pennies. Therefore, Danny had $1.75 in quarters and $0.10 in pennies. This also totals $1.85.

Solve the following problems. Use equations to represent the situation in each problem and tables to help you organize the information.

10. Vladimir's age is double the sum of Enzo's age and 4. Two years ago he was 3 times Enzo's age. Find their present ages.

11. Mrs. Wasylyk, Mrs. Chiu, and Mrs. Nepoose were discussing investments on the drive to work. They discovered that they all had purchased shares in the same company. Mrs. Chiu's initial investment was five times more than Mrs. Wasylyk's. Mrs. Nepoose's was three times the difference between Mrs. Wasylyk's and $20. The difference between what Mrs. Chiu had invested and what Mrs. Nepoose invested was $1000 more than what Mrs. Wasylyk had invested.

a. What was each lady's initial investment?

b. If Mrs. Wasylyk bought 400 shares, what was the cost per share?

c. If Mrs. Wasylyk's investment is now worth $1300, how much has each share increased in price?

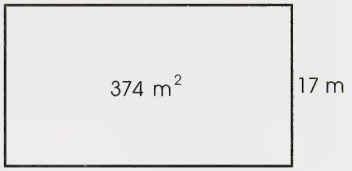
d. What is the percent increase in the value of Mrs. Wasylyk's shares? Round your answer to the nearest percent.

12. A vending machine, when emptied by the attendant at the end of the week, contained the following distribution of coins: twice as many dimes as nickels, four times as many quarters as dimes, and ten more loonies than quarters. If the attendant collected $133, how many of each type of coin were in the machine?

Check your answers by turning to the Appendix, page 146.

Rearranging Formulas

Given the information about a rectangular enclosure, what is the length of the enclosure? To obtain the length, divide 374 by 17. You should get a length of 22 m.



Recall that the formula for the area of a rectangle is $A = \ell \times w$. The method of finding the length of any rectangle can be represented by the area formula rearranged as $w = \frac{A}{\ell}$. The rearranged formula indicates that you divide the area by the length to find the width—just as you did for the rectangular enclosure.

There are many situations in which rearranging a known formula to find the value of a certain variable is convenient or even necessary.

The following example illustrates two ways of finding the missing value. You should be familiar with both methods. Both methods involve rearrangement.

Example

The formula $d = vt$ is used to find the distance travelled when the average rate of speed and the time travelled are known. In this formula, d is the distance travelled (in km), v is the average rate of speed (in km/h), and t is the time travelled (in h).

If Mrs. Rhodes drove 400 km in 5 h, what was her average rate of speed?

Solution

Method 1

Step 1: Solve for v.

$$d = vt$$

$$\frac{d}{t} = \frac{vt}{t}$$

$$\frac{d}{t} = v$$

$$v = \frac{d}{t}$$

Step 2: Substitute the known values.

If $d = 400$ and $t = 5$, then

$$\begin{aligned} v &= \frac{d}{t} \\ &= \frac{400}{5} \\ &= 80 \end{aligned}$$

Mrs. Rhodes travelled at an average rate of speed of 80 km/h.

Method 2

Substitute the known values, and solve for v.

If $d = 400$ and $t = 5$, then

$$\begin{aligned} d &= vt \\ 400 &= v \times 5 \\ 5v &= 400 \\ \frac{5v}{5} &= \frac{400}{5} \\ v &= 80 \end{aligned}$$

Mrs. Rhodes travelled at an average rate of speed of 80 km/h.

13. Rearrange each of the following formulas to indicate the value of the variable shown in parenthesis. In the rearranged formula, the variable in parenthesis should be isolated on the left of the equal sign.

a. $C = \pi d \quad (d)$

b. $P = a + b + c \quad (a)$

c. $A = \frac{1}{2}ab \quad (b)$

d. $V = \ell wh \quad (\ell)$

e. $A = \frac{1}{2}(b_1 + b_2) \times h \quad (h)$

14. Find the value of the missing variable for each of the following situations. Round your answer to the nearest tenth where necessary.

a. Using the formula from question 13.a., find d if $C = 27.004$ cm.

b. Using the formula from question 13.b., find a if $P = 48.5$ cm, $b = 7.6$ cm, and $c = 19.4$ cm.

c. Using the formula from question 13.c., find b if $A = 63.6$ cm^2 and $a = 10.6$ cm.

d. Using the formula from question 13.d., find ℓ if $V = 336$ cm^3, $w = 8.4$ cm, and $h = 3.2$ cm.

e. Using the formula from question 13.e., find h if $A = 126$ cm^2, $b_1 = 12$ cm, and $b_2 = 18$ cm.

15. Explain how $d = vt$ and $v = \frac{d}{t}$ are related.

16. The Langfield School hockey team is sponsoring their zone hockey tournament. The formula $C = 50 + 0.1n$ is used to find the cost of printing hockey tickets. In this formula, C is the cost (in dollars) of printing and n is the number of tickets printed.

If the cost of printing the tickets was \$350, how many tickets were printed?

Check your answers by turning to the Appendix, page 148.

Now Try This

Use one of the problem-solving strategies to solve the following problem.

17. Elsie wants to get a photograph enlarged so that the length and width are twice those of the original. She knows the original cost of the print is \$1.60 and is suprised when she is told the cost of the enlargement will be \$6.40. Elsie assumed the cost would be twice the original or \$3.20. As an employee of the photograph shop, how would you explain to Elsie in terms of the areas of the two photographs why the cost is \$6.40?

Check your answer by turning to the Appendix, page 150.

Looking Back

In this activity, you solved a variety of equations. You wrote equations that represented problems and then solved the equation to determine the answer to the problem. You also rearranged formulas.

You will see many equations in science and mathematics courses because equations are a convenient way to represent situations.

18. In your journal, write a letter to a friend explaining why formulas are important.

Check your answers by turning to the Appendix, page 150.

Activity 3: Solving More-Complicated Equations Containing Fractions

So far in this section, you have solved some simple equations containing fractions. Some equations containing fractions can be more complicated than others. These equations will require more steps to solve.

Example

Craig, James and Derek shared a package of gummy bears. Craig took $\frac{1}{2}$ of the package, James took $\frac{1}{4}$ of the package, and Derek took $\frac{1}{8}$ of the package. If there were 7 gummy bears left after they took their shares, how many gummy bears were in the package in the beginning?

Solution

Step 1: Write the equation.

Let n be the total number of gummy bears in the package. Thus, Craig took $\frac{1}{2}n$ (or $\frac{n}{2}$) gummy bears, James took $\frac{1}{4}n$ (or $\frac{n}{4}$) gummy bears, and Derek took $\frac{1}{8}n$ (or $\frac{n}{8}$) gummy bears.

$$\frac{n}{2}+\frac{n}{4}+\frac{n}{8}+7=n$$

Step 2: Find the LCM (lowest common multiple) of the denominators.

The LCM of 2, 4, and 8 is 8.

Step 3: Multiply each side of the equation by 8 to clear the denominators; then solve for n.

$$8\left(\frac{n}{2}+\frac{n}{4}+\frac{n}{8}+7\right)=8(n)$$

$$\overset{4}{\cancel{8}}\left(\frac{n}{\cancel{2}_{1}}\right)+\overset{2}{\cancel{8}}\left(\frac{n}{\cancel{4}_{1}}\right)+\overset{1}{\cancel{8}}\left(\frac{n}{\cancel{8}_{1}}\right)+8(7)=8n$$

$$4n+2n+n+56=8n$$

$$7n+56=8n$$

$$7n+56-7n=8n-7n$$

$$56=n$$

$$n=56$$

Check

$\frac{1}{2}\times 56=28$ $\quad$ $\frac{1}{4}\times 56=14$ $\quad$ $\frac{1}{8}\times 56=7$

$28+14+7+7=56$

It checks out.

Therefore, there were 56 gummy bears in the package in the beginning.

The following examples illustrate some of the different, more complicated equations involving fractions.

Example

Solve the following equation; then verify the solution.

$$\frac{m+2}{4}=\frac{3m-6}{8}$$

Solution

Step 1: Find the LCM of the denominators. The LCM of 4 and 8 is 8.

Step 2: Multiply each side of the equation by 8 to clear the denominators; then solve for m.

$$\overset{2}{\cancel{8}}\left(\frac{m+2}{\cancel{4}_{1}}\right)=\overset{1}{\cancel{8}}\left(\frac{3m-6}{\cancel{8}_{1}}\right)$$

$$2m+4=3m-6$$
$$2m+4-4=3m-6-4$$
$$2m=3m-10$$
$$2m-3m=3m-10-3m$$
$$-m=-10$$
$$m=10$$

Check

Substitute 10 for m in the original equation.

LS	RS
$\frac{m+2}{4}$	$\frac{3m-6}{8}$
$=\frac{10+2}{4}$	$=\frac{3(10)-6}{8}$
$=\frac{12}{4}$	$=\frac{30-6}{8}$
$=3$	$=\frac{24}{8}$
	$=3$

LS = RS

Example

Solve the following equation; then verify the solution.

$$\frac{2m}{3}+\frac{m}{4}=\frac{11}{6}$$

Solution

Step 1: Find the LCM of the denominators.

The LCM of 3, 4, and 6 is 12.

Step 2: Multiply each side of the equation by 12 to clear the denominators; then solve for m.

$$12\left(\frac{2m}{3}+\frac{m}{4}\right)=12\left(\frac{11}{6}\right)$$

$$\overset{4}{\cancel{12}}\left(\frac{2m}{\cancel{3}_{1}}\right)+\overset{3}{\cancel{12}}\left(\frac{m}{\cancel{4}_{1}}\right)=\overset{2}{\cancel{12}}\left(\frac{11}{\cancel{6}_{1}}\right)$$

$$8m+3m=22$$

$$11m=22$$

$$\frac{11m}{11}=\frac{22}{11}$$

$$m=2$$

Check

LS	RS
$\frac{2m}{3}+\frac{m}{4}$	$\frac{11}{6}$
$=\frac{2(2)}{3}+\frac{2}{4}$	
$=\frac{4}{3}+\frac{2}{4}$	
$=\frac{16}{12}+\frac{6}{12}$	
$=\frac{22}{12}$	
$=\frac{11}{6}$	

LS = RS

1. Solve the following equations. Verify your answers with mental mathematics or your calculator.

a. $\frac{3m}{9}=\frac{7}{6}$

b. $\frac{y+2}{5}=\frac{2y+1}{9}$

c. $\frac{2d}{5}+\frac{d}{10}=\frac{15}{2}$

d. $\frac{2n}{5}=\frac{n-6}{4}$

e. $\frac{n}{2}+\frac{n}{3}=n-\frac{1}{8}$

f. $\frac{5}{n+3}=\frac{9}{2n}$

g. $\frac{a}{2}+\frac{a}{3}+\frac{a}{4}+\frac{a}{6}=15$

2. When Mr. Johnson died, his estate was divided according to the instructions in his last will and testament. His wife, Arlene, received $\frac{1}{2}$ of the estate; his son, Brad, received $\frac{1}{4}$, his niece, Jill, received $\frac{1}{5}$ of the estate; and the remainder was divided equally among his 3 favourite charities. If each charity received $3400, what was the total value of the estate and how much did each person receive?

3. If $\frac{2}{3}$ of a number, decreased by $\frac{1}{2}$ is equal to $\frac{5}{6}$, what is the number?

Check your answers by turning to the Appendix, page 151.

Solving Equations with the Variable in the Denominator

Up to this point, you have worked with equations with the variable in the numerator. Now, work through an example where the variable is in the denominator.

Example

Solve the following equation; verify the solution.

$$\frac{3}{a}=\frac{5}{4}$$

Solution

Step 1: Find the LCM of the denominators.

The LCM of a and 4 is $4a$.

Step 2: Multiply each side of the equation by $4a$ to clear the denominators, and then solve for a.

$$4\cancel{a}\left(\frac{3}{\cancel{a}}\right)=\cancel{4}a\left(\frac{5}{\cancel{4}}\right)$$

$$12=5a$$

$$\frac{5a}{5}=\frac{12}{5}$$

$$a=\frac{12}{5} \text{ or } 2.4$$

Check

LS	RS
$\frac{3}{a}$	$\frac{5}{4}$
$=\frac{3}{\frac{12}{5}}$	
$=\cancel{3}^{1}\times\frac{5}{\cancel{12}_{4}}$	
$=\frac{5}{4}$	
LS = RS	

LS	RS
$\frac{3}{a}$	$\frac{5}{4}$
$=\frac{3}{2.4}$	
$=\frac{3\times10}{2.4\times10}$	
$=\frac{30}{24}$	
$=\frac{5}{4}$	
LS = RS	

Now, you will work with equations that have a variable in the denominator.

4. Solve the following equations.

a. $\frac{5}{a}=10$

b. $\frac{7}{a}+1=8$

c. $\frac{3}{r}=\frac{5}{8}$

d. $\frac{7}{x}-2=\frac{4}{5}$

5. Nadia can pick 32 containers of wild blueberries in 5 h.

   a. At this rate, how many containers can she pick in 8 h?

   b. How long will it take her to pick 112 containers at this rate?

6. The number 24 is divided into two parts such that the ratio of the two parts is 5 to 7. If the larger part is 4 more than the smaller part, find the two parts.

Check your answers by turning to the Appendix, page 154.

Looking Back

You have now solved some of the most complicated equations. You have seen that the same method of balancing equations is used for both simple and complex equations.

7. Copy the following equation into your journal; then solve and verify this equation. Begin by multiplying by 18 to avoid fractions. This will remind you how to avoid fractions.

$$\frac{x}{3} - \frac{x}{6} = 2 - \frac{x}{9}$$

Check your answer by turning to the Appendix, page 156.

Conclusion

In this section, you solved equations and verified the solutions using models and paper and pencil. You also used equation-solving skills to solve problems and evaluate formulas.

You found that maintaining balance is very important in solving equations. For this reason, a balance scale is used to visualize equations. The scale is a reminder that whenever you perform an operation on one side of an equation, you must perform the same operation on the other side of the equation.

Learning to solve equations is like learning to ride a bicycle. Maintaining balance becomes easier with practice.

Assignment

Turn to Assignment Booklet 3B and complete the assignment for Section 2.

Module Summary

THE EDMONTON JOURNAL

In this module, you investigated a branch of mathematics called algebra. You became familiar with the language of algebra, and you translated English phrases into mathematical expressions and vice versa. You then worked with the formulas for many geometric shapes, calculating both volume and surface area. To help you visualize algebraic expressions and equations, you used learning aids to model them. You solved equations by adding, subtracting, multiplying, and dividing, and you used equations to solve problems.

Mathematics is a universal language and the meanings of algebraic expressions and equations are understood by mathematicians all over the world.

The language of mathematics can be compared to the language of mimes. Mathematicians use variables, numerals, and operation signs to describe patterns and relations. Mimes use facial expressions, gestures, and body language to tell a story.

Final Module Assignment

Turn to Assignment Booklet 3B and complete the final module assignment for Module 3.

Appendix

Problem-Solving Strategies
Glossary
Suggested Answers
Credits
Formula Sheet
Cut-out Learning Aids

Problem-Solving Strategies

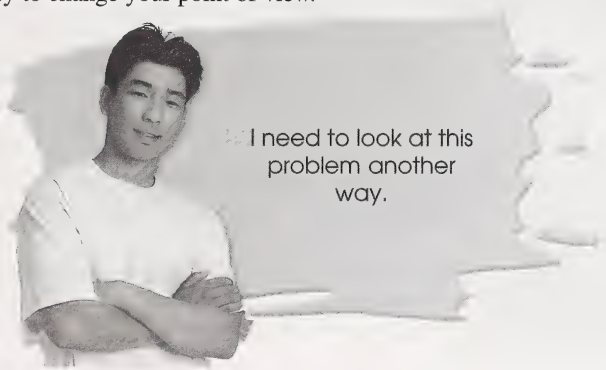
Be sure to study the following problem-solving strategies and use them when you encounter problems in this course.

Whichever method of problem solving you may choose to use, consider the following:

- Show that you understand the problem by showing all the steps needed for finding the answer.
- Draw and neatly label any diagram, graph, or chart that may help you answer the problem.
- Write a concluding sentence that answers the question being asked.

Changing Your Point of View

Sometimes you find that you cannot solve a particular problem because of your "mind set." Perhaps you made an assumption about the problem that is incorrect. If your attempts to solve a problem are not successful, it is often helpful to try to change your point of view.

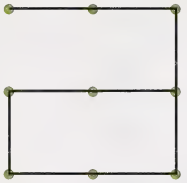


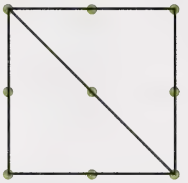
Example

Draw four line segments through the following nine dots without lifting your pencil from the paper and without retracing your path.

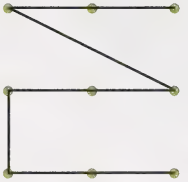
Solution

Step 1: You make several attempts at solving the problem.


5 line segments

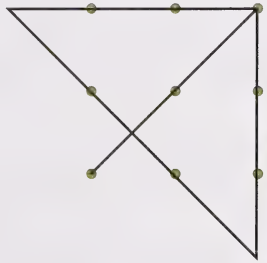

5 line segments

5 line segments


5 line segments

Step 2: You realize you are blocked in your attempts. You have made the assumption that the dots must be connected in a way similar to a child's dot-to-dot picture.

In order to solve the problem, you must change your point of view and realize that the line segments may lie outside the confines of the dots.



4 line segments

Using Objects

Objects can be used to help you solve a problem.

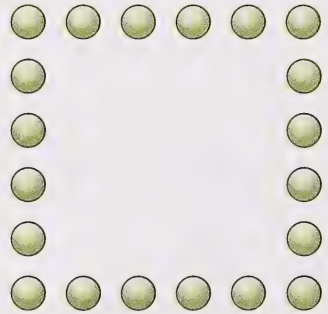
Example

To enclose a square lot with a fence, 20 posts are used. If the posts are placed 3 m apart, what is the length of one side of the lot?

Solution

Step 1: Use objects such as blocks, checkers, or coins to represent the posts.

The advantage of using objects to represent the posts is that you can easily rearrange them.



Step 2: Count the number of spaces between the posts on one side of the square.

There are five spaces between the six posts on one side of the square.

Step 3: Calculate the length of one side of the square.

$$5 \times 3 = 15$$

The length of one side of the lot is 15 m.

Example

Four days before the day after tomorrow was Monday. What day of the week is it today?

Solution

Step 1: Use paper cups to represent the days. The cups could be labelled as shown.

Step 2: Begin at the cup representing the day after tomorrow and count back four days. That day was Monday.

If Monday was the day before yesterday, yesterday was Tuesday, and today is Wednesday.

Using Diagrams

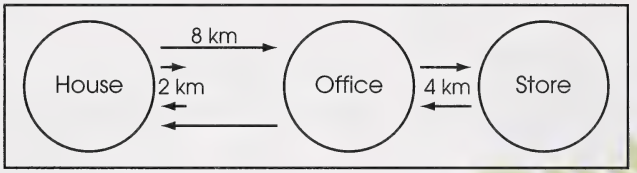
You can use sketches to solve problems.

Example

Chris's office is 8 km from his home. Yesterday morning he drove 2 km before he realized that he had forgotten his briefcase. He returned home to get his briefcase and then drove to his office. At noon, Chris drove 4 km to a store. He then went back to the office for the rest of the afternoon. At the end of the day, he drove straight home. How far did he drive yesterday?

Solution

Step 1: Draw and label a diagram to help you understand the problem.



Step 2: Calculate the distance Chris drove.

$$2+2+8+4+4+8=28$$

Chris drove 28 km.

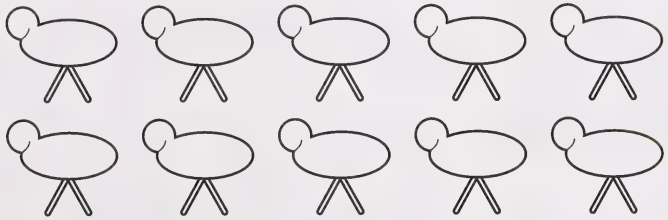
Example

There are 10 animals in a barnyard. Some are chickens and the rest are sheep. Ruth counts 28 legs. How many chickens and how many sheep are there? **Hint:** Chickens have 2 legs and sheep have 4 legs.

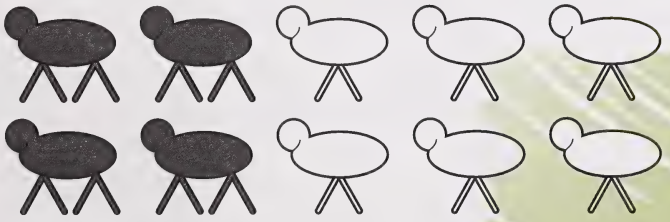
Solution

Draw a diagram to help you solve the problem.

Step 1: Every animal has at least 2 legs. So, draw a diagram of 10 animals, each of which has at least 2 legs.



Step 2: The group of animals in the problem had 28 legs. So, you need to add 8 legs to the drawing. Add the legs in pairs to the sketch of each animal.



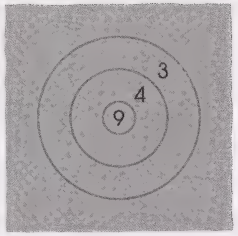
There are 4 sheep and 6 chickens.

Making an Organized List

Some problems require you to list all the possible solutions. It is important to make an organized list so that you do not miss any possibilities.

Example

Julio and his friends like playing a game called Bull's Eye with lawn darts. They draw a target in the dirt and assign values to the three regions of the target.



If Julio throws three darts and all three hit the target, how many different point totals are possible?

Solution

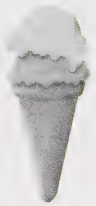
Begin with the highest possible score and list all the possibilities in order.

$9+9+9=27$ ← highest possible score
$9+9+4=22$
$9+9+3=21$
$9+4+4=17$
$9+4+3=16$
$9+3+3=15$
$4+4+4=12$
$4+4+3=11$
$4+3+3=10$
$3+3+3=9$ ← lowest possible score

There are 10 possible point totals.

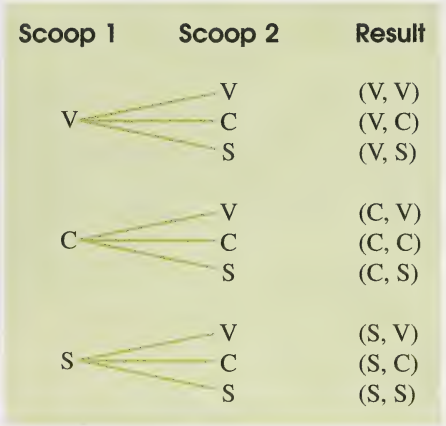
Example

At an ice-cream parlour there are 3 flavours of ice cream: chocolate, vanilla, and strawberry. How many different double-scoop ice-cream cones are possible? **Hint:** A chocolate-vanilla cone is different from a vanilla-chocolate one. Order makes a difference.



Solution

Draw a tree diagram to help you make an organized list. Use V for vanilla, C for chocolate, and S for strawberry.



Nine different double-scoop ice-cream cones are possible.

Using Venn Diagrams

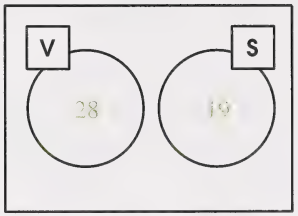
A type of diagram that is helpful in some problems is a Venn diagram. In this special kind of diagram, circles are used to represent groups of people, animals, or objects that have certain characteristics. The positioning of the circles in relation to one another represents relationships among these groups. These diagrams were named after an English mathematician by the name of John Venn (1834–1923).

Example

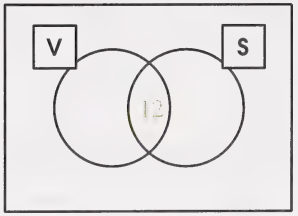
There are 28 girls on the volleyball team and 19 girls on the swim team. If 12 girls belong to both teams, how many girls belong only to the swim team?

Solution

Step 1: Represent the members of the volleyball team with a circle. Represent the members of the swim team with another circle.



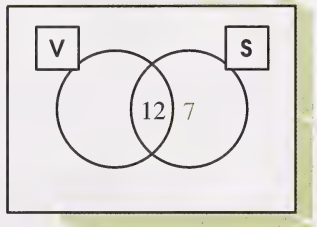
Step 2: Represent the members of both teams by the intersection of the two circles.



Step 3: Now, calculate the number of girls who belong only to the swim team.

$$19 - 12 = 7$$

So, 7 girls belong only to the swim team.

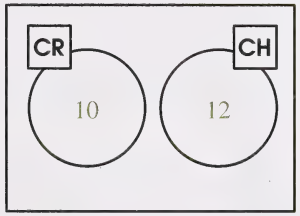


Example

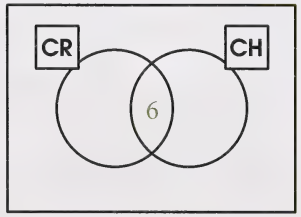
Reilly bought some plates at a garage sale. He discovered that 10 plates were cracked, 12 plates were chipped, 6 plates were both chipped and cracked, and 2 plates were neither chipped nor cracked. How many plates did Reilly buy?

Solution

Step 1: The problem states that 10 plates were cracked (CR) and 12 plates were chipped (CH). Represent the cracked plates with a circle. Represent the chipped plates with a second circle.

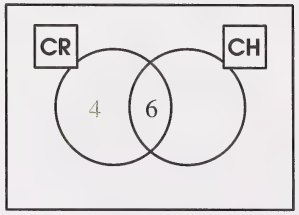


Step 2: The problem states that 6 plates were both chipped and cracked. Represent the cracked and chipped plates with the intersection of the two circles.



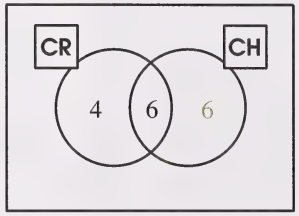
Step 3: Calculate the number of plates that were only cracked.

$$10 - 6 = 4$$

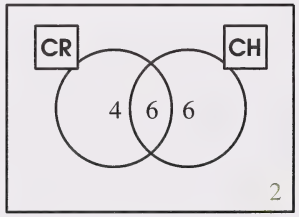


Step 4: Calculate the number of plates that were only chipped.

$$12 - 6 = 6$$



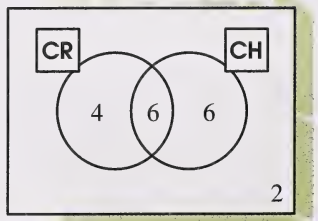
Step 5: The problem states that 2 plates were neither chipped nor cracked. Represent the undamaged plates with the region outside the circles.



Step 6: Calculate the total number of plates that were bought.

$$4 + 6 + 6 + 2 = 18$$

So, 18 plates were bought.



Example

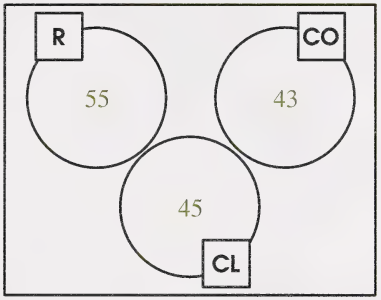
In a survey of 100 people, the following information was collected:

- 43 people like country music
- 55 like rock music
- 45 like classical music
- 8 people like all three forms
- 15 people like both country and classical music
- 25 like both rock and country music
- 20 like both rock and classical music

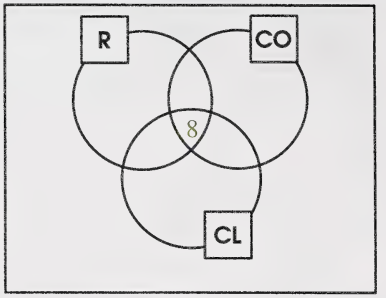
How many people do not like any of these forms of music?

Solution

Step 1: The problem states that 55 people like rock music (R), 43 like country music (CO), and 45 like classical music (CL). Use three circles to represent these preferences.

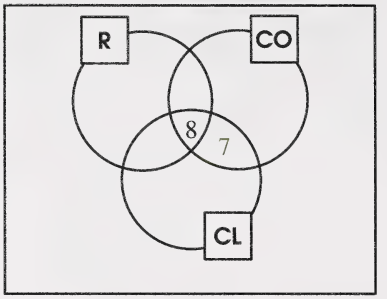


Step 2: The problem states that 8 people like all three forms. The intersection of the three circles shows the people who like all three forms.



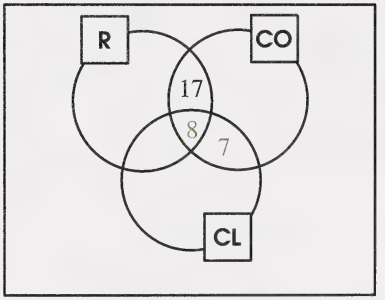
Step 3: The problem states that 15 people like both country and classical music. Calculate the number of people who like country and classical music, but not rock music.

$$15 - 8 = 7$$



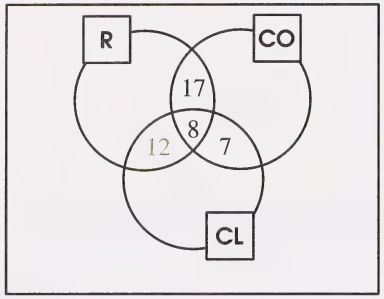
Step 4: The problem states that 25 people like both rock and country music. Calculate the number of people who like both country and rock music, but not classical music.

$$25 - 8 = 17$$



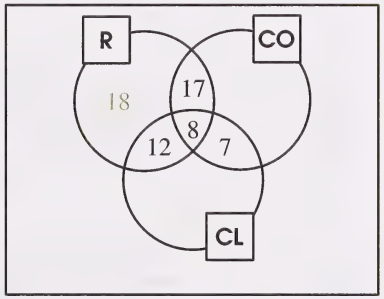
Step 5: The problem states that 20 people like both rock and classical music. Calculate the number of people who like both rock and classical music, but not country music.

$$20 - 8 = 12$$



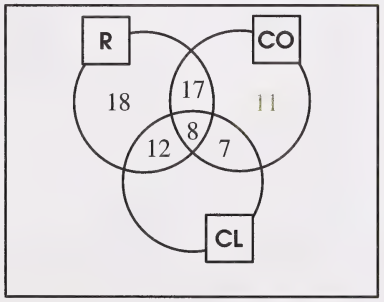
Step 6: Now that you know the number of people who like rock music and any other form of music, calculate the number of people who only like rock.

$$55 - (17 + 8 + 12) = 18$$



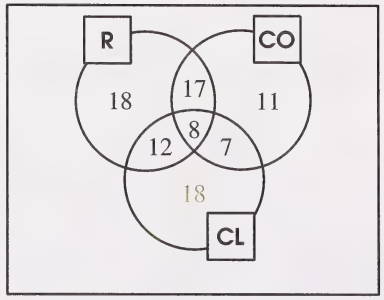
Step 7: Now that you know the number of people who like country music and any other form of music, calculate the number of people who only like country music.

$$43 - (17 + 8 + 7) = 11$$



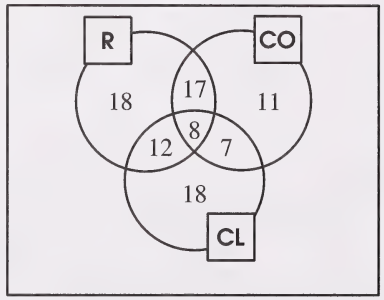
Step 8: Now that you know the number of people who like classical music and any other form of music, calculate the number of people who only like classical music.

$$45 - (12 + 8 + 7) = 18$$



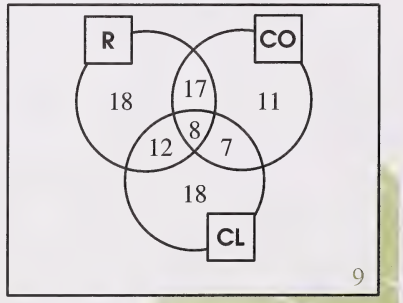
Step 9: Now, you can calculate the total number of people who like rock, country, or classical music.

$$18 + 17 + 8 + 12$$
$$+ 11 + 7 + 18 = 91$$



Step 10: The problem states that 100 people were surveyed. So, you can calculate the number of people who do not like any of these forms of music.

$$100 - 91 = 9$$



Of the 100 people surveyed, 9 people do not like any of these forms of music.

Making a Table

You can use a table to organize information.

Example

Hannah works at her mother's store after school. She earns 20¢ for each customer that she assists and 35¢ for each bag that she packs. One day she did 19 jobs and earned $4.85. How many bags did she pack and how many customers did she assist that day?

Solution

Write down all possible combinations of 19 jobs until you find a combination that totals $4.85. Use a table to organize the information.

In the first column, put 1 bag packed and 18 customers assisted. Calculate the money received for that.

$$1 \times 35¢ = \$\ .35$$
$$18 \times 20¢ = \$3.60$$
$$\text{Total} \quad \$3.95$$

Then complete the table for other combinations.

Packing	1	2	3	4	5	6	7
Assisting	18	17	16	15	14	13	12
Earned	$3.95	$4.10	$4.25	$4.40	$4.55	$4.70	$4.85

correct guess

Hannah packed 7 bags and assisted 12 customers that day.

Guessing, Checking, and Revising

Many mathematical problems can be solved by systematic trial or guessing, checking, and revising.

Example

Shelley opened her math textbook and noticed that the product of the numbers of the two open pages was 2970. To what pages was the book opened? **Hint:** The left-hand page number of a book is even; the right-hand page number is odd.

Solution

Method 1: Using Paper and Pencil

Step 1: Note the conditions that need to be met.

- The left-hand page number must be even.
- The right-hand page number is one more than the left-hand page number.
- The product of the numbers must be 2970.

Step 2: Make a guess; check your guess; and revise if necessary.

It is helpful to organize your guesses in a table.

Left-Hand Page Number	50	52	54
Right-Hand Page Number	51	53	55
Product of Page Numbers	2550	2756	2970

correct guess

So, the book was opened to pages 54 and 55.

Method 2: Using a Computer

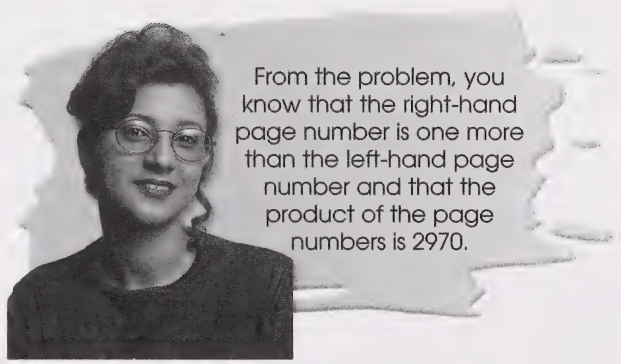
Use a computer and a spreadsheet program to help you solve the problem.

Step 1: Label the cells in the spreadsheet. **Hint:** The cell address at the top of the spreadsheet shows which cell is active. There is also a border around the active cell. You don't type directly in this cell; instead, you use the entry bar at the top of the spreadsheet.

A5 ✗ ✓ Product of page numbers

	A	B
1	**Book Problem**	
2		
3	Left-hand page number	
4	Right-hand page number	
5	Product of page numbers	
6		

Step 2: Enter either a number or a formula for each of the cells.

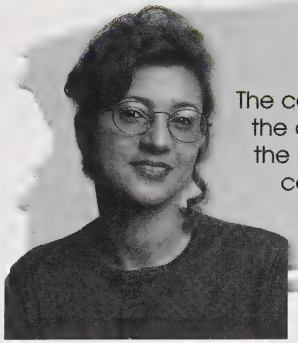


Here is what you enter:

- Begin with cell B3 and enter a guess of 50.
- Next, enter "=B3+1" in cell B4. This means the number in the B4 cell will be one more than the number in the B3 cell.
- Finally, enter "=B3*B4" in cell B5. This means that the number in the B5 cell will be the product of the numbers in the B3 and B4 cells.

B5 ✗ ✓ =B3*B4

	A	B
1	**Book Problem**	
2		
3	Left-hand page number	50
4	Right-hand page number	51
5	Product of page numbers	2550
6		



The computer performs the calculations and the results appear in cells B4 and B5.

Step 3: Check your initial guess, and revise it if necessary. The product is 2970, not 2550; therefore, revise your guess.

Delete the number in the B3 cell and enter a guess of 54.

B3 ✗ ✓ 54

	A	B
1	**Book Problem**	
2		
3	Left-hand page number	54
4	Right-hand page number	55
5	Product of page numbers	2970
6		

The computer performs the calculations and different results appear in cells B4 and B5.

Step 4: Check your initial guess, and revise it if necessary. The product is 2970.

So, the book was opened to pages 54 and 55.

Acting Out a Problem

For some problems, you may find it helpful to physically act out the problem situation.

Example

How many cards does each boy have?

Solution

Step 1: Ask yourself these questions:

- What would happen if Jon gave Matt a card? The first condition to be met is that they must have an equal number of cards after the exchange.

- What would happen if Matt gave Jon a card? The second condition to be met is that Jon will have twice as many cards as Matt after the exchange.

Step 2: Make a guess and act out the problem. If your guess does not fit the conditions, revise your guess and repeat the process.

It is helpful to organize your guesses in a table.

Matt's cards	1	2	3	4	5
Jon's cards	3	4	5	6	7

correct guess

So, Matt has 5 cards and Jon has 7 cards.

Note

If Jon gives Matt a card, each boy will have an equal number of cards after the exchange, 6.

If Matt gives Jon a card, Jon will have twice as many cards as Matt after the exchange. (Jon will have 8 cards and Matt will have 4 cards.)

Working Backwards

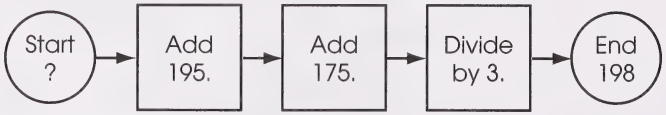
In mathematics, you are usually given information and then asked to find the answer. Sometimes, however, you are given the answer and then asked to find a piece of information. In cases like these, you need to work backwards.

Example

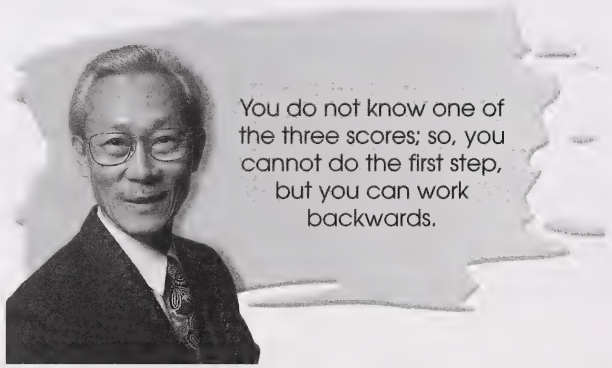
Ross plays in a bowling league. His average in the league is 198. Today Ross scored 195 and 175 in his first two games. What must he score in his third game to maintain his average?

Solution

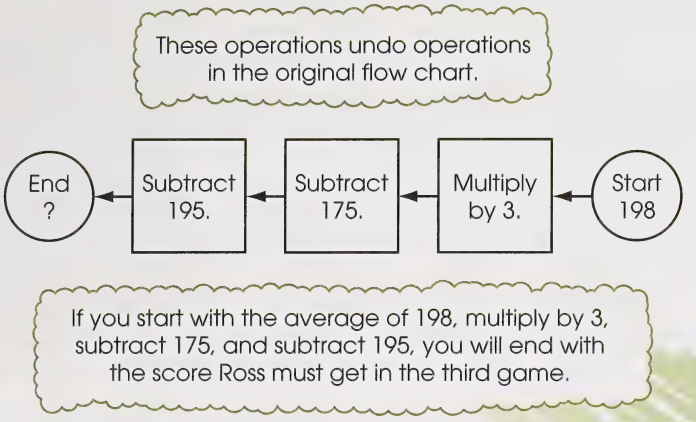
Step 1: Consider how the problem would be done if you worked forwards. This flow chart shows the sequence of the steps.



You start with the score of the third game, add each of the scores of the other two games, divide by 3, and end with the average of 198.



Step 2: Use a reverse flow chart to work backwards.

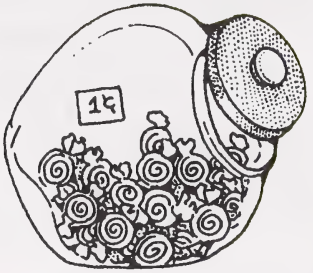


Ross must score 224 in the last game to maintain his average.

Note: You can check to see if 224 is the correct third score by using the flow chart in Step 1.

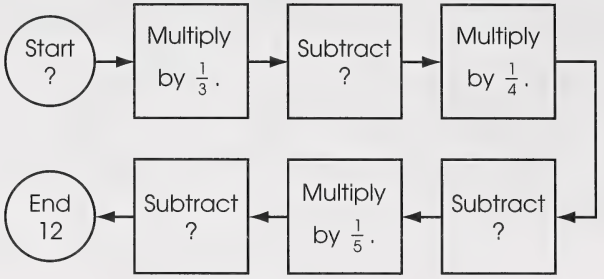
Example

Ruth has a jar of candies. She gives Raschid one-third of her candies. She then gives Steven one-fourth of the remaining candies. Finally, she gives one-fifth of the candies that she has left to Rachel. If Ruth has 12 candies left at the end, how many did she have at the beginning?



Solution

Step 1: Use a flow chart to show the order of operations.

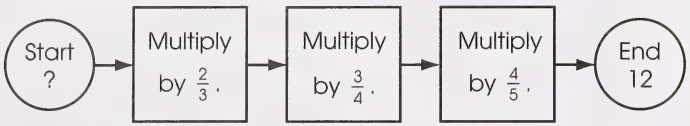


This flow chart is **not** helpful because you do not know how much to subtract in the second, fourth, and sixth boxes.

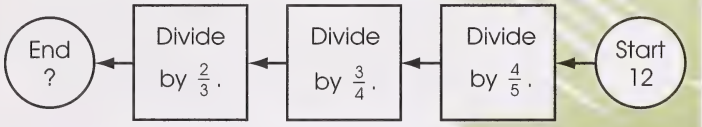
Knowing the following relationships will help you draw a more complete flow chart.

- Giving away one-third of the candies is the same as multiplying the number of candies by two-thirds. $1-\frac{1}{3}=\frac{2}{3}$
- Giving away one-fourth of the remaining candies is the same as multiplying by three-fourths. $1-\frac{1}{4}=\frac{3}{4}$
- Giving away one-fifth of the remaining candies is the same as multiplying by four-fifths. $1-\frac{1}{5}=\frac{4}{5}$

The following flow chart is more useful.



Step 2: Use a reverse flow chart to work backwards.



Ruth had 30 candies at the beginning.

Simplifying a Problem

Are you sometimes confused about which operations to perform in a problem because the numbers in the problem are very large or because the problem seems very complicated? There are strategies you can use to simplify a problem.

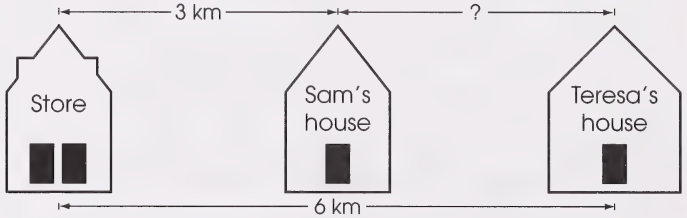
Example

The average distance from the Sun to Mars is 228 000 000 km, and the average distance from the Sun to Earth is 150 000 000 km. What is the average distance between Earth and Mars?

Solution

Step 1: If you find the large numbers overwhelming, make up a related but simpler problem.

Sam and Teresa both live east of the store. Sam lives 3 km from the store and Teresa lives 6 km from the store. How far is it from Sam's house to Teresa's house?



$6 - 3 = 3$

It is 3 km from Sam's house to Teresa's house.

Step 2: Once you understand the related but simpler problem, you can solve the original problem.

$$228\,000\,000 - 150\,000\,000 = 78\,000\,000$$

The average distance from Earth to Mars is 78 000 000 km.

Finding and Applying a Pattern

To solve some problems, it is often helpful to use a related but simpler problem to discover a pattern.

Example

There are 20 students in a room. If every student shakes hands with every other student in the room, how many handshakes are exchanged?

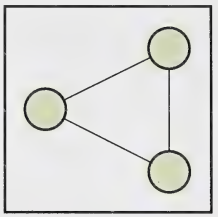
Solution

Step 1: Try solving the problem with 1, 2, 3, 4, or 5 students.

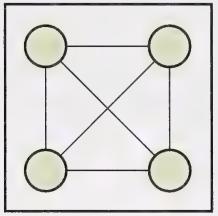
- If there is 1 student, no handshakes can be exchanged (as illustrated in the following diagram).

- If there are 2 students, 1 handshake can be exchanged (as illustrated in the following diagram).

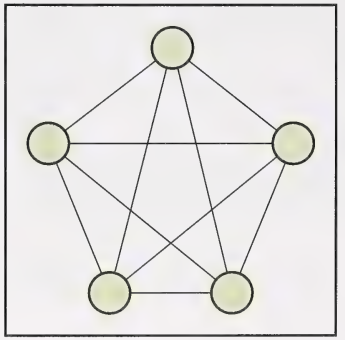
- If there are 3 students, 3 handshakes can be exchanged (as illustrated in the following diagram).



- If there are 4 students, 6 handshakes can be exchanged (as illustrated in the following diagram).



- If there are 5 students, 10 handshakes can be exchanged (as illustrated in the following diagram).

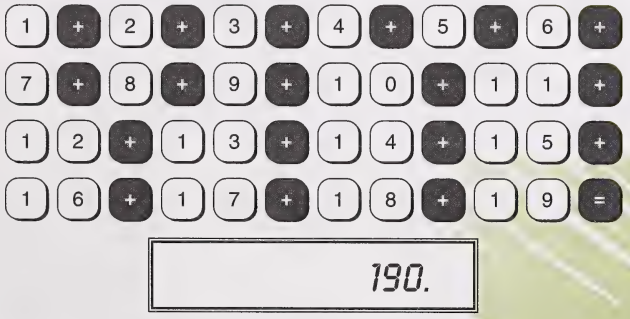


Step 2: Look for a pattern.

People	Handshakes	Pattern
1	0	
2	1	+1
3	3	+2
4	6	+3
5	10	+4

Step 3: Apply the pattern.

A calculator is helpful for applying the pattern. Go up to 19, because each student will shake the hand of 19 others.

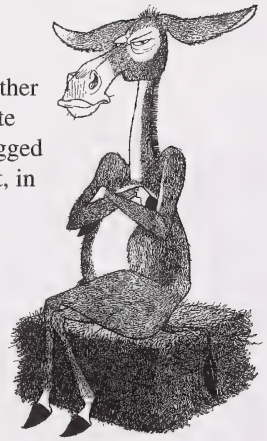


There are a total of 190 handshakes exchanged.

Using Elimination

Detectives are skilful at eliminating clues and solving mysteries. You can become skilful at using elimination to solve problems too.

Example



Five animals had a race over a short distance. Neither the donkey nor the coyote beat the lion. The coyote beat the rabbit, but not the ostrich. The donkey lagged behind the rabbit. If the ostrich was not the fastest, in what order did the animals finish the race?

Solution

Step 1: Decide what clues are given. The clues indicate the following:

- The donkey and the coyote did not finish first because they did not beat the lion.
- The rabbit did not finish first because the coyote beat it.
- The ostrich was faster than the coyote.
- The rabbit was faster than the donkey.
- The ostrich did not finish first because it was not the fastest.

The clues can be shown on a table.

	Donkey	Coyote	Rabbit	Ostrich	Lion
First	✗	✗	✗	✗	
Second					
Third					
Fourth					
Fifth					

Step 2: Use elimination and reconsider the clues.

By elimination, it is clear that the lion was first. Now, reconsider the clues:

- The rabbit did not finish second because the coyote beat it.
- The coyote did not finish second because the ostrich was faster.
- The donkey did not finish second because it lagged behind the rabbit.

The additional clues can be shown in the table.

	Donkey	Coyote	Rabbit	Ostrich	Lion
First	✗	✗	✗	✗	✓
Second	✗	✗	✗		✗
Third					✗
Fourth					✗
Fifth					✗

Step 3: Use elimination and reconsider the clues.

By elimination, it is clear that the ostrich finished second. Now, reconsider the clues:

- The rabbit did not finish third because the coyote beat it.
- The donkey did not finish third because it lagged behind the rabbit.

The additional clues can be shown in the table.

	Donkey	Coyote	Rabbit	Ostrich	Lion
First	✗	✗	✗	✗	✓
Second	✗	✗	✗	✓	✗
Third	✗		✓	✓	✗
Fourth				✗	✗
Fifth				✓	✗

Step 4: Use elimination and reconsider the clues.

By elimination, it is clear that the coyote finished third. Now, reconsider the clues.

Because the donkey lagged behind the rabbit, the rabbit was fourth and the donkey was fifth.

	Donkey	Coyote	Rabbit	Ostrich	Lion
First	✗	✗	✗	✗	✓
Second	✗	✗	✗	✓	✗
Third	✗	✓	✗	✗	✗
Fourth	✗	✗	✓	✗	✗
Fifth	✓	✗	✗	✗	✗

So, the lion was first, the ostrich was second, the coyote was third, the rabbit was fourth, and the donkey was last.

Using Truth Tables

A strategy you can use to solve some logic problems is making truth tables. Truth tables are similar to elimination tables.

Example

Detective Beagle is investigating a robbery in the forest. Charlie Chipmunk and Sammy Squirrel each deny being the robber. Harold Hare blames Charlie Chipmunk. Detective Beagle knows two of the animals are lying and one is telling the truth. Who is the robber?

Solution

Step 1: Assume that Charlie Chipmunk was the robber.

- Charlie Chipmunk said he didn't do it. If Charlie Chipmunk was the robber, this is a false statement. Make a truth table and put **F** under Charlie's name.

Charlie Chipmunk	Sammy Squirrel	Harold Hare
F		

- Sammy Squirrel denied doing it. If Charlie Chipmunk was the robber, this is a true statement. Put **T** under Sammy's name in the truth table.

Charlie Chipmunk	Sammy Squirrel	Harold Hare
F	**T**	

- Harold Hare blamed Charlie Chipmunk. If Charlie Chipmunk was the robber, this is a true statement. Put **T** under Harold's name in the truth table.

Charlie Chipmunk	Sammy Squirrel	Harold Hare
F	T	**T**

Detective Beagle knows two animals are lying. So, Charlie is not the robber.

Step 2: Assume that Sammy Squirrel was the robber.

- Charlie Chipmunk denied doing it. If Sammy Squirrel was the robber, this is a true statement. Put **T** under Charlie's name in the truth table.

Charlie Chipmunk	Sammy Squirrel	Harold Hare
F	T	T
T		

- Sammy Squirrel denied doing it. If Sammy Squirrel was the robber, this is a false statement. Put **F** under Sammy's name in the truth table.

Charlie Chipmunk	Sammy Squirrel	Harold Hare
F	T	T
T	**F**	

- Harold Hare blamed Charlie Chipmunk. If Sammy Squirrel was the robber, this is a false statement. Put **F** under Harold's name in the truth table.

Charlie Chipmunk	Sammy Squirrel	Harold Hare
F	T	T
T	F	**F**

Detective Beagle knows two of the animals are lying and one is telling the truth.

So, Sammy Squirrel was the robber.

Using an Equation

A strategy you can use to solve some problems is to write and solve an equation.

Example

Twelve less than a number is -3. Find the number.

Solution

Step 1: Let the unknown be equal to a variable. Let the number be n.

Step 2: Write an equation.

$$n - 12 = -3$$

Twelve less than a number is -3.

Step 3: Solve the equation.

$$\begin{array}{r} n - 12 = -3 \\ +12 \quad +12 \\ \hline n \quad = \quad 9 \end{array}$$

The number is 9.

Example

The second of two numbers is 7 times the first. The sum of the two numbers is 32. Find the numbers.

Solution

Step 1: Write statements for the unknowns. Use only one variable.

Let n be the first number and let $7n$ be the second number.

The second of two numbers is 7 times the first.

Step 2: Write an equation.

$$n + 7n = 32$$

The sum of the two numbers is 32.

Step 3: Solve the equation.

$$\begin{aligned} n + 7n &= 32 \\ 8n &= 32 \\ \frac{8n}{8} &= \frac{32}{8} \\ n &= 4 \end{aligned}$$

The first number is 4.

Step 4: Find the second number if the first number is 4.

$$\begin{aligned} 7n &= 7 \times 4 \\ &= 28 \end{aligned}$$

The second number is 28.

So, the numbers are 4 and 28.

Example

Yvonne has 20 more nickels than dimes in her piggy bank. If the total value of the nickels and dimes is $8.50, how many coins of each type does she have?

Solution

Step 1: Write statements for the unknowns. Use only one variable.

Let the number of dimes be n.

The value of the nickels, the number of dimes, and the value of the dimes must also be represented by algebraic expressions.

A table can be used to organize the information.

Type of Coin	Number of Coins	Value in Cents
Dimes	n	$10n$
Nickels	$n + 20$	$5(n + 20)$
Total		850

The value of the dimes is 10 times the number of dimes.

The value of the nickels is 5 times the number of nickels.

Yvonne has 20 more nickels than dimes.

This is given.

Step 2: Write an equation.

$$5(n+20)+10n=850$$

Step 3: Solve the equation.

$$\begin{aligned} 5(n+20)+10n &= 850 \\ 5n+100+10n &= 850 \\ 15n+100 &= 850 \\ -100 \quad &-100 \\ \hline 15n \quad &= 750 \\ \frac{15n}{15} &= \frac{750}{15} \\ n &= 50 \end{aligned}$$

Yvonne has 50 dimes in her piggy bank.

Step 4: Find the number of nickels if the number of dimes is 50.

$$\begin{aligned} n+20 &= 50+20 \\ &= 70 \end{aligned}$$

Yvonne has 70 nickels in her piggy bank.

So, Yvonne has 50 dimes and 70 nickels in her piggy bank.

Glossary

algebra: the branch of mathematics that describes basic arithmetic relations

algebraic equation: an equation containing a variable

e.g., $2a = 6$

algebraic expression: a combination of numerals, variables, and/or other mathematical symbols

e.g., $3a + 2$

apothem: the perpendicular distance from the centre of a regular polygon to a side

base (of a polyhedron): the face or pair of faces from which the height of the polyhedron is measured

circumference: the perimeter or distance around a circle

$C = 2\pi r$, where r is the radius

coefficient: a number multiplying a variable in a mathematical expression or equation

composite shape: an object composed of a combination of two or more basic shapes

conditional equation: an equation that is only true for certain values of the variable

consecutive integers: integers written in sequential order

coordinates: the numbers in an ordered pair that show the position of a point on a grid

dependent variable: the variable that depends on the values chosen for the independent variable

diameter: the distance across a circle measured through the centre

edge: a line segment formed by the intersection of two faces

evaluating: finding the value of a mathematical expression

equation: any mathematical statement that contains an equal sign

face: one of the flat surfaces of a polyhedron

formula: an equation showing how one quantity is related to one or more other quantities

height (of a polyhedron): the vertical distance from a vertex to the base

independent variable: the variable you control

integer: a number from the set $\{\ldots, -3, -2, -1, 0, 1, 2, \ldots\}$

inspection method: a method of solving algebraic equations by looking at them

mathematical expression: a phrase that involves numbers and operations

net: a two-dimensional figure that can be assembled to create a three-dimensional shape

ordered pair: a pair for which the order is important

order of operations: a set of rules that mathematicians have agreed upon to find the value of a mathematical expression

origin: the point at which the x-axis and the y-axis meet

perimeter: the distance around a figure

polyhedron: (pl. **polyhedra**) a three-dimensional shape that has only flat surfaces

prism: a polyhedron

problem: a task for which the method of finding the answer (as well as the answer itself) is not immediately known

regular polyhedron: a polyhedron where all faces are identical regular polygons and the same number of faces and the same number of edges meet at each vertex

solution: the value of the variable that makes an equation true

solving an equation: the process of finding the value of a variable that makes the equation true

variable: a quantity represented by a symbol that may have many values

variable expression: a mathematical expression that uses one or more variables; also called an **algebraic expression**

vertex: (pl. **vertices**) a point in a polyhedron where three or more edges meet

volume: a measurement of how much space an object occupies; measured in cubic units

x-axis: the horizontal axis on a graph

y-axis: the vertical axis on a graph

Suggested Answers

Section 1: Activity 1

1. Answers may vary. Sample answers are given.

a. one less than the product of two and a number

b. twice the difference between a number and one

c. four added to the product of five and a number

d. the sum of four and five, times a number

e. six minus the product of three and a number

f. six minus three, times a number

2. **a.** $2n+3$ **b.** $4n-5$ **c.** $2(n+5)$ **d.** $4(n-1)$

3. Variables may differ. Each of the following answers uses the first letter of the key word from the phrase as the variable.

a. $a+2$ **b.** $2d$ **c.** $5p$

d. $a+12$ **e.** $\frac{2}{3}s$ **f.** $\frac{e}{2}$

g. $3(a+j)$ **h.** $w-e$ **i.** $\frac{1}{2}r+2m$

Looking Back

4.

Operation Symbol	Words and Phrases
+	plus, increased by, more than, added to, the sum of
−	minus, decreased by, diminished by, less than, subtract, the difference between
×	times, multiplied by, the product of
÷	divided by, the quotient of

You may add new words or phrases for each symbol as you encounter them.

Section 1: Activity 2

1. a. $C = 12.00 + 0.50t$
$= 12.00 + 0.50(1)$
$= 12.00 + 0.50$
$= 12.50$

It costs $12.50 to rent skis for 1 h.

b. $C = 12.00 + 0.50t$
$= 12.00 + 0.50(2)$
$= 12.00 + 1.00$
$= 13.00$

It costs $13.00 to rent skis for 2 h.

c. $C = 12.00 + 0.50t$
$= 12.00 + 0.50(3)$
$= 12.00 + 1.50$
$= 13.50$

It costs $13.50 to rent skis for 3 h.

d. $C = 12.00 + 0.50t$

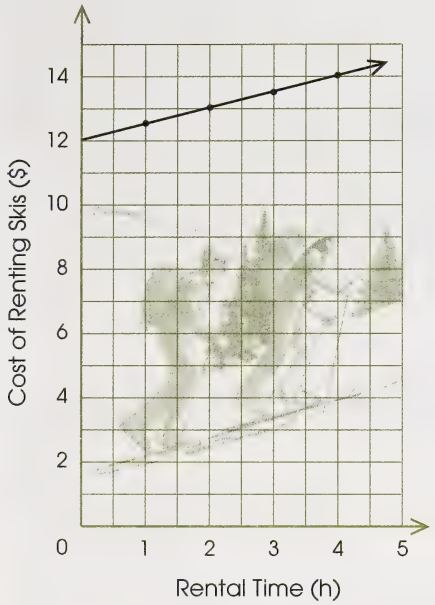
$= 12.00 + 0.50(4)$

$= 12.00 + 2.00$

$= 14.00$

It costs \$14.00 to rent skis for 4 h.

e.



f. As the time increases, the cost of renting skis increases.

2. a. **Step 1:** Represent the unknown quantities by variables.

Let r be the reward (in dollars) and let n be the number of marks earned.

Step 2: Write a formula for Reward Option 1.

$r = 4n - 5$

Step 3: Write a formula for Reward Option 2.

$r = 3n$

Step 4: Write a formula for Reward Option 3.

$r = 10 + 1n$

$= 10 + n$

The 1 (one) in $1n$ is generally not written.

b.

Reward Option 1

Number of Marks	Reward ($)
0	−5
1	−1
2	3
3	7
…	…

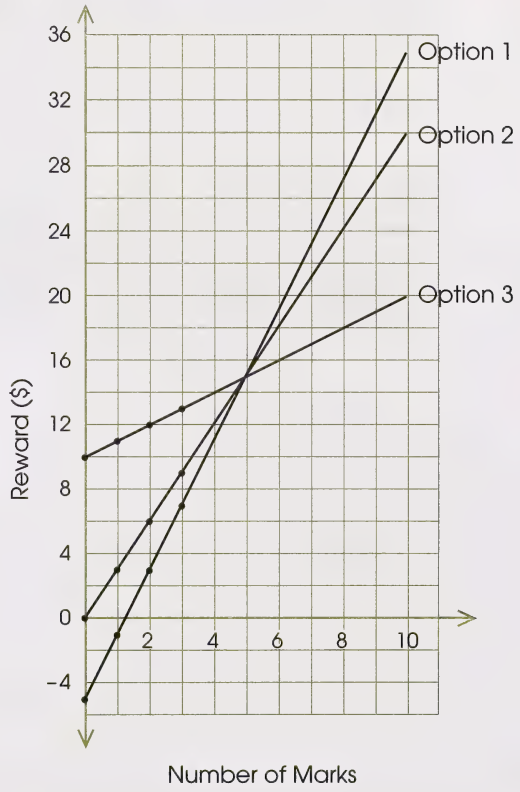
Reward Option 2

Number of Marks	Reward ($)
0	0
1	3
2	6
3	9
...	...

Reward Option 3

Number of Marks	Reward ($)
0	10
1	11
2	12
3	13
...	...

c. Graph of Reward Options 1, 2, and 3.



There are no arrowheads on each line because the highest possible number of marks is 10.

d. Nicholas should pick Option 3 if he expects to earn no marks because he would get a reward of \$10. If he picks Option 2, he would get \$0; and if he picks Option 1, he would owe his mother \$5.

e. Nicholas should pick Option 1 if he expects to get all ten questions correct because he would get a reward of \$35. If he picks Option 2, he would only get \$30; and if he picks Option 3, he would only get \$20.

f. All the options pay the same amount if Nicholas earns five marks. Each option pays \$15.

3. a.

Mass of Roast (kg)	Cooking Time (min)
1	40
2	80
3	120
4	160
5	200

b. $c = 40m$

c. $c = 40m$
$= 40(4.5)$
$= 180$

It will take 180 min or 3 h to cook a 4.5-kg roast.

4. a. Noel did not put an arrowhead on the graph because the plant will not continue to grow forever. An arrow suggests the line goes on to infinity.

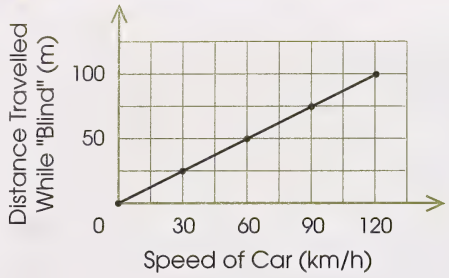
b.

Time (days)	Height (cm)
0	4
1	5
2	6
3	7
4	8
5	9

c. $h = t + 4$

d. Noel wanted to determine the relationship between the height of the plant and the time since it was planted. Since Noel wanted to describe how height depends on time, time is the independent variable and height is the dependent variable.

5. a.



b. The formula shows the relationship or pattern between the two types of data. Finding the relationship takes some trial and error. Look at the second pair of numbers in the chart: 25 and 30. The relationship could be $25+5=30$, or stated with the variables, $d+5=s$. This equation can be written as $d=s-5$. Another possible relationship between 25 and 30 is $25=\frac{5}{6}\times 30$. Stated with the variables, this would be $d=\frac{5}{6}s$.

Now, look at the next pair of numbers, 50 and 60. Test the formulas describing the previous pair of numbers to see if they are true for this pair.

Test $d+5=s$.

LS	RS
$d+5$	s
$=50+5$	$=60$
$=55$	

LS $\neq$ RS

Test $d=\frac{5}{6}s$.

LS	RS
d	$\frac{5}{6}s$
$=50$	$=\frac{5}{6}(60)$
	$=50$

LS = RS

So, the formula $d=\frac{5}{6}s$ fits for two of the pairs of numbers. Test it with the other pairs.

The formula is $d=\frac{5}{6}s$.

c. $d=\frac{5}{6}s$

$=\frac{5}{\cancel{6}_{1}}\left(\cancel{96}^{16}\right)$

$=80$

The person will travel 80 m while "blind."

6. a.

Rectangles with an Area of 16 Square Units

Length (units)	Width (units)	Perimeter (units)
1	16	34
2	8	20
4	4	16
8	2	20
16	1	34

b. The rectangle with a length of 1 unit and a width of 16 units and the rectangle with a length of 16 units and a width of 1 unit have the greatest perimeter.

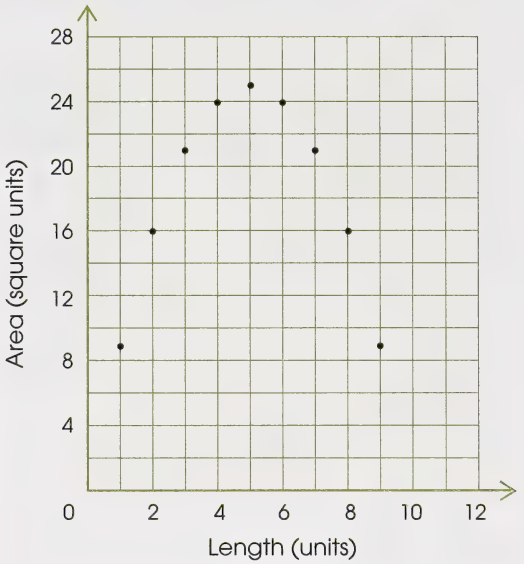
c. The rectangle with a length and width of 4 units has the least perimeter.

7. a.

Rectangles with a Perimeter of 20 Units

Length (units)	Width (units)	Area (square units)
1	9	9
2	8	16
3	7	21
4	6	24
5	5	25
6	4	24
7	3	21
8	2	16
9	1	9

b. Rectangles with a Perimeter of 20 Units



You determined the area based on various values chosen for the length. You can see that length is the independent variable and should be placed on the horizontal axis. Area is the dependent variable and should be placed on the vertical axis.

c. Rectangles with a Perimeter of 20 Units

The open dots on the *x*-axis indicate there is no rectangle with a length of 0 units or 10 units that has a perimeter of 20 units.

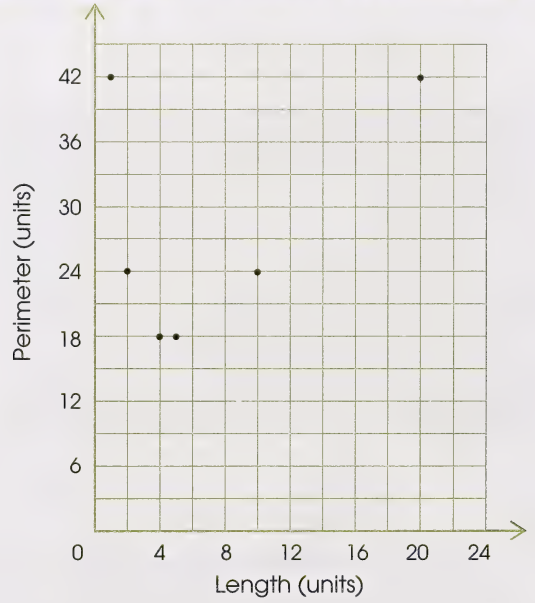
d. Starting at the origin, the area increases as the length increases. The maximum area is the rectangle with a length of 5 units; this is the turning point. After this point, the area decreases as the length increases.

e. The rectangle that is 5 units by 5 units has the greatest area. This rectangle is a square.

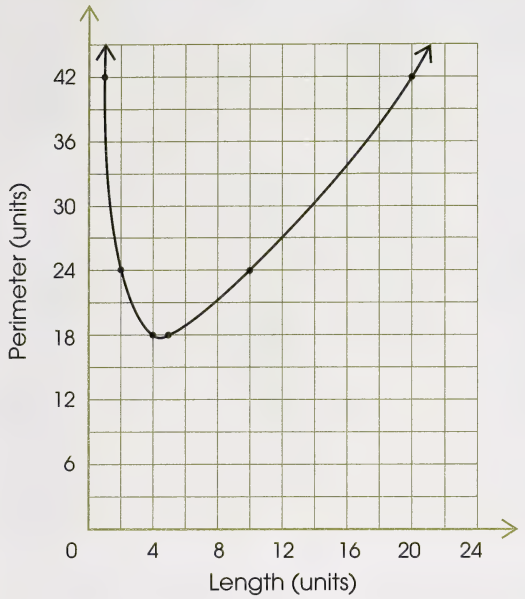
8. a. **Rectangles with an Area of 20 Square Units**

Length (units)	Width (units)	Perimeter (units)
1	20	42
2	10	24
4	5	18
5	4	18
10	2	24
20	1	42

b. Rectangles with an Area of 20 Square Units



c. Rectangles with an Area of 20 Square Units



d. At first, the perimeter decreases as the length of the rectangle increases. The least perimeter occurs when a rectangle has a length of about 4.5 units; this is the turning point. After this point, the perimeter increases as the length increases.

9. You may list the lengths and widths of rectangles with an area of 144 m^2 to find the one with the least perimeter.

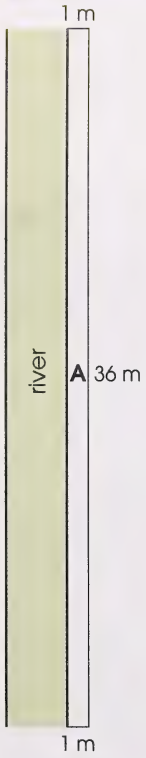
Rectangles with an Area of 144 m^2

Length (m)	Width (m)	Perimeter (m)	
1	144	290	
2	72	148	
3	48	102	
4	36	80	
6	24	60	
8	18	52	
9	16	50	
12	12	48	← minimum perimeter
16	9	50	
18	8	52	
24	6	60	
36	4	80	
48	3	102	
72	2	148	
144	1	290	

The rectangle with a length of 12 m and a width of 12 m has the least perimeter.

So, to use the least fencing, the dog run should be 12 m by 12 m.

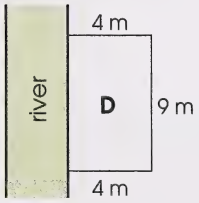
10. a.



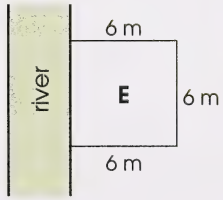
$1+1+36=38$ m

$2+2+18=22$ m

$12+3+3=18$ m



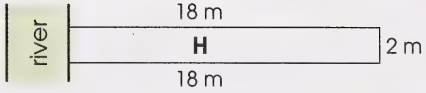
$4+4+9=17$ m



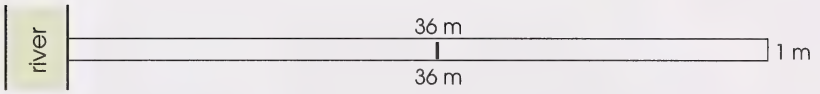
$6+6+6=18$ m

$9+9+4=22$ m

$12+12+3=27$ m



$18+18+2=38$ m



$36+36+1=73$ m

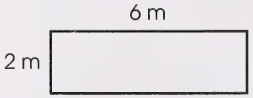
b. The most economical design is pen D; it would require only 17 m of fencing.

c. The most expensive design is pen I; it would require 73 m of fencing.

11. You can use the guess, check, and revise strategy to solve the problem.

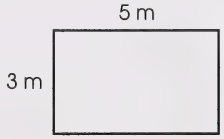
The pen will be 10 m by 11 m.

12. The following are different shapes of pens with 16 m of fencing.



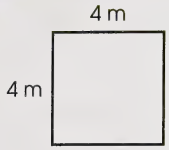
$A=6\times2$

$=12$ m^2



$A=5\times3$

$=15$ m^2



$A = 4 \times 4$

$= 16 \text{ m}^2$

$C = 2\pi r$

$16 = 2\pi r$

$r = \dfrac{16}{2\pi}$

$\doteq 2.55$

$A = \pi r^2$

$\doteq \pi \times (2.55)^2$

$\doteq 20 \text{ m}^2$

A circular pen gives the largest area.

Looking Back

13. a. If rectangles with the same area are being compared, the rectangle with the smallest perimeter would **be the square with the given area**.

b. If rectangles with the same perimeter are being compared, the rectangle with the largest area would **be the square with the given perimeter**.

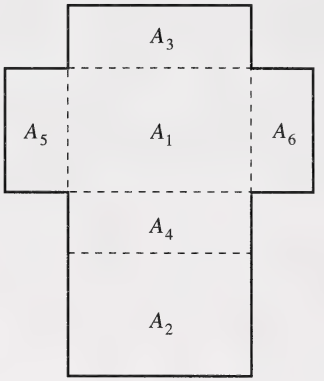
14. a. Let d be the number of days the car is rented; let k be the number of kilometres the car is driven; and let C be the cost (in dollars) of renting a car.

Therefore, the formula is $C = 45d + 0.25k$.

b. The cost of renting a car depends on the number of days rented and the number of kilometres driven.

Section 1: Activity 3

1. a. Step 1: Represent the surface area of the box as the sum of the areas of its six faces. **Note:** $A_1 = A_2$, $A_3 = A_4$, and $A_5 = A_6$



Step 2: Find the area of A_1 and A_2. **Note:** Each rectangle is $17.8 \text{ cm} \times 12.4 \text{ cm}$.

$A_1 = \ell w$

$= 17.8(12.4)$

$= 220.72$ $\quad \therefore A_2 = 220.72$

The area of A_1 and A_2 together is $2 \times 220.72 = 441.44 \text{ cm}^2$.

Step 3: Find the area of A_3 and A_4. **Note:** Each rectangle is $17.8 \text{ cm} \times 6.0 \text{ cm}$.

$$\begin{aligned} A_3 &= \ell w \\ &= 17.8(6.0) \\ &= 106.8 \qquad \therefore A_4 = 106.8 \end{aligned}$$

The area of A_3 and A_4 together is $2 \times 106.8 = 213.6 \text{ cm}^2$.

Step 4: Find the area of A_5 and A_6. **Note:** Each rectangle is $12.4 \text{ cm} \times 6.0 \text{ cm}$.

$$\begin{aligned} A_5 &= \ell w \\ &= 12.4(6.0) \\ &= 74.4 \qquad \therefore A_6 = 74.4 \end{aligned}$$

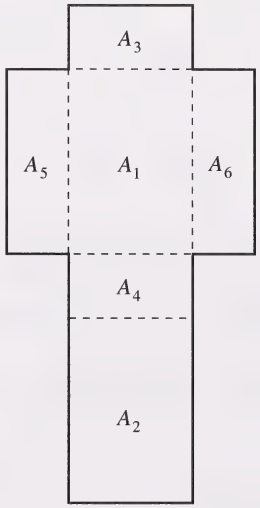
The area of A_5 and A_6 together is $2 \times 74.4 = 148.8 \text{ cm}^2$.

Step 5: Find the surface area of the box.

$$\begin{aligned} S &= (A_1 + A_2) + (A_3 + A_4) + (A_5 + A_6) \\ &= 441.44 + 213.6 + 148.8 \\ &= 803.84 \end{aligned}$$

The surface area is 803.84 cm^2.

b. Step 1: Represent the surface area as the sum of the areas of its six faces. **Note:** $A_1 = A_2$, $A_3 = A_4$, and $A_5 = A_6$



Step 2: Find the area of A_1 and A_2. **Note:** Each rectangle is $8.0 \text{ cm} \times 4.5 \text{ cm}$.

$$\begin{aligned} A_1 &= \ell w \\ &= 8.0(4.5) \\ &= 36 \qquad \therefore A_2 = 36 \end{aligned}$$

The area of A_1 and A_2 together is $2 \times 36 = 72 \text{ cm}^2$.

Step 3: Find the area of A_3 and A_4. **Note:** Each rectangle is $13.0\text{ cm} \times 8.0\text{ cm}$.

$$\begin{aligned} A_3 &= \ell w \\ &= 13.0(8.0) \\ &= 104 \qquad \therefore A_4 = 104 \end{aligned}$$

The area of A_3 and A_4 together is $2 \times 104 = 208\text{ cm}^2$.

Step 4: Find the area of A_5 and A_6. **Note:** Each rectangle is $13.0\text{ cm} \times 4.5\text{ cm}$.

$$\begin{aligned} A_5 &= \ell w \\ &= 13.0(4.5) \\ &= 58.5 \qquad \therefore A_6 = 58.5 \end{aligned}$$

The area of A_5 and A_6 together is $2 \times 58.5 = 117\text{ cm}^2$.

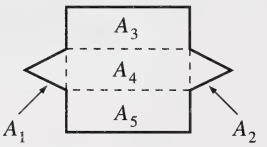
Step 5: Find the surface area of the box.

$$\begin{aligned} S &= (A_1 + A_2) + (A_3 + A_4) + (A_5 + A_6) \\ &= 72 + 208 + 117 \\ &= 397 \end{aligned}$$

The surface area is 397 cm^2.

2. Step 1: Represent the surface area as the sum of the areas of its five faces.

Note: $A_1 = A_2$ and $A_3 = A_4 = A_5$



Step 2: Find the area of A_1 and A_2. **Note:** Each triangle has a base of 3.0 cm and a height of 2.6 cm.

$$\begin{aligned} A_1 &= \frac{1}{2}bh \\ &= \frac{1}{2}(3.0)(2.6) \\ &= 3.9 \qquad \therefore A_2 = 3.9 \end{aligned}$$

The area of A_1 and A_2 together is $2 \times 3.9 = 7.8\text{ cm}^2$.

Step 3: Find the area of A_3, A_4, and A_5. **Note:** Each rectangle is $3.0\text{ cm} \times 16.7\text{ cm}$.

$$\begin{aligned} A_3 &= \ell w \\ &= 16.7(3.0) \\ &= 50.1 \qquad \therefore A_4 = A_5 = 50.1 \end{aligned}$$

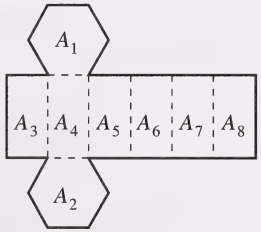
The area of A_3, A_4, and A_5 together is $3 \times 50.1 = 150.3\text{ cm}^2$.

Step 4: Find the surface area of the box.

$$\begin{aligned} S &= (A_1 + A_2) + (A_3 + A_4 + A_5) \\ &= 7.8 + 150.3 \\ &= 158.1 \end{aligned}$$

The surface area is 158.1 cm^2.

3. Step 1: Represent the surface area as the sum of the areas of its eight faces. **Note:** $A_1 = A_2$ and $A_3 = A_4 = A_5 = A_6 = A_7 = A_8$



Step 2: Find the area of A_1 and A_2. **Note:** Each hexagon has 6 sides of 7.5 cm, and the apothem is 6.5 cm.

$$\begin{aligned} A_1 &= \frac{nsa}{2} \\ &= \frac{6(7.5)(6.5)}{2} \\ &= \frac{292.5}{2} \\ &= 146.25 \end{aligned} \qquad \therefore A_2 = 146.25$$

The area of A_1 and A_2 together is $2 \times 146.25 = 292.5 \text{ cm}^2$.

Step 3: Find the area of A_3, A_4, A_5, A_6, A_7, and A_8. **Note:** Each rectangle is $7.5 \text{ cm} \times 5.8 \text{ cm}$.

$$\begin{aligned} A_3 &= \ell w \\ &= 7.5(5.8) \\ &= 43.5 \end{aligned}$$

$$\therefore A_4 = A_5 = A_6 = A_7 = A_8 = 43.5$$

The area of A_3, A_4, A_5, A_6, A_7, and A_8 together is $6 \times 43.5 = 261 \text{ cm}^2$.

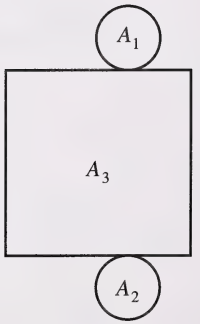
Step 4: Find the surface area of the box.

$$\begin{aligned} S &= (A_1 + A_2) + (A_3 + A_4 + A_5 + A_6 + A_7 + A_8) \\ &= 292.5 + 261 \\ &= 553.5 \end{aligned}$$

The surface area is 553.5 cm^2.

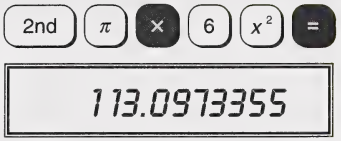
4. Represent the surface area of each cylinder as the sum of the areas of their three faces.

Note: $A_1 = A_2$



a. **Step 1:** Find the area of A_1 and A_2. **Note:** Each circular base has a diameter of 12 cm and a radius of 6 cm.

$$\begin{aligned} A_1 &= \pi r^2 \\ &= \pi\left(6^2\right) \\ &\doteq 113.1 \end{aligned}$$

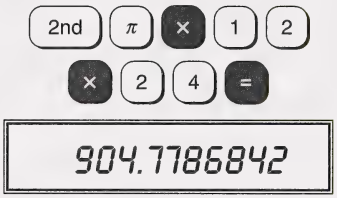


$\therefore A_2 \doteq 113.1$

The area of A_1 and A_2 together is about 226.2 m^2.

Step 2: Find the area of A_3 (the curved surface). **Note:** The curved surface can be flattened to make a rectangle. The length of the rectangle is the circumference, *C*, of the base. The width of the rectangle is the height, *H*, of the can.

$$\begin{aligned} \therefore A_3 &= \ell w \\ &= CH \\ &= (\pi d)(24) \\ &= \pi(12)(24) \\ &\doteq 904.8 \end{aligned}$$



The area of A_3 is about 904.8 cm^2.

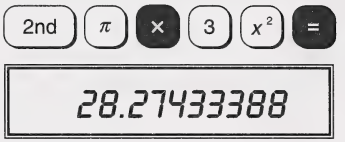
Step 3: Find the surface area of the cylinder.

$$\begin{aligned} S &= (A_1 + A_2) + A_3 \\ &\doteq 226.2 + 904.8 \\ &\doteq 1131.0 \end{aligned}$$

The surface area of the can is about 1131 cm^2.

b. **Step 1:** Find the area of A_1 and A_2. **Note:** Each circular base has a diameter of 6 m and a radius of 3 m.

$$\begin{aligned} A_1 &= \pi r^2 \\ &\doteq \pi\left(3^2\right) \\ &\doteq 28.3 \end{aligned}$$

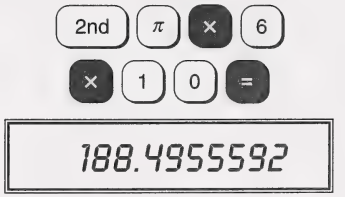


$\therefore A_2 \doteq 28.3$

The area of A_1 and A_2 together is about 56.6 m^2.

Step 2: Find the area of A_3 (the curved surface). **Note:** The length of the rectangle is πd, and the width of the rectangle is 10 m.

$$\begin{aligned} A_3 &= \ell w \\ &= (\pi d)(10) \\ &= \pi(6)(10) \\ &\doteq 188.5 \end{aligned}$$



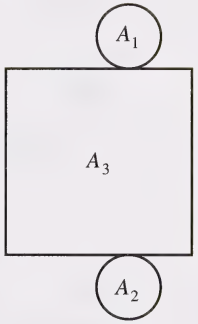
The area of A_3 is about 188.5 m^2.

Step 3: Find the surface area of the cylinder.

$$\begin{aligned} S &= (A_1 + A_2) + A_3 \\ &\doteq 56.6 + 188.5 \\ &\doteq 245.1 \end{aligned}$$

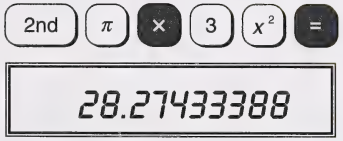
The surface area of the cylinder is about $245\ \text{m}^2$.

5. Step 1: Represent the surface area as the sum of the areas of its three faces. **Note:** $A_1 = A_2$



Step 2: Find the area of A_1 and A_2.

$$\begin{aligned} A_1 &= \pi r^2 \\ &\doteq \pi (3)^2 \\ &\doteq 28.3 \end{aligned}$$

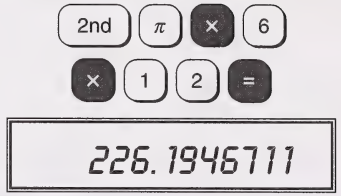


$\therefore A_2 \doteq 28.3$

The area of A_1 and A_2 together is about $56.6\ \text{cm}^2$.

Step 3: Find the area of A_3 (the curved side). **Note:** The length of this rectangle is πd, and the width of this side is 12 cm.

$$\begin{aligned} A_3 &= \ell w \\ &= (\pi d)(12) \\ &= \pi (6)(12) \\ &\doteq 226.2 \end{aligned}$$

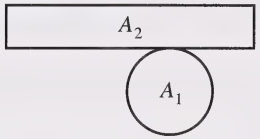


Step 4: Find the surface area.

$$\begin{aligned} S &= (A_1 + A_2) + A_3 \\ &\doteq 56.6 + 226.2 \\ &\doteq 282.8 \end{aligned}$$

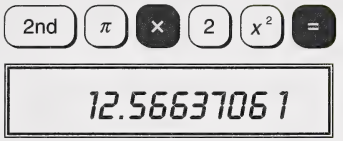
The total surface area is about $283\ \text{cm}^2$.

6. Step 1: Represent the surface area of the liner as the sum of the areas of its two faces.



Step 2: Calculate the area of A_1. **Note:** The circle has a radius of 2 m.

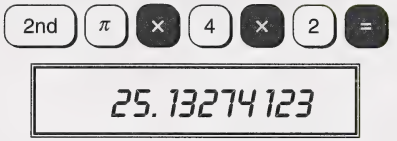
$$A_1 = \pi r^2$$
$$= \pi(2)^2$$
$$\doteq 12.57$$



The area of A_1 is about 12.57 m^2.

Step 3: Calculate the area of A_2. **Note:** The rectangle has a length of πd (the circumference of the cylindrical pool). The width is 2 m.

$$A_2 = \ell w$$
$$= (\pi d)(2)$$
$$= \pi(4)(2)$$
$$\doteq 25.14$$



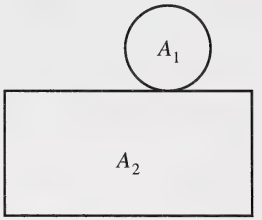
The area of A_2 is 25.14 m^2.

Step 4: Calculate the surface area of the liner.

$$S = A_1 + A_2$$
$$\doteq 12.57 + 25.14$$
$$\doteq 37.71$$

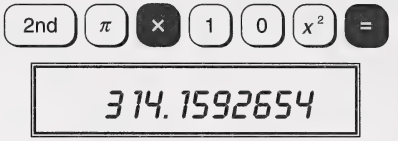
The surface area of the liner is about 37.7 m^2.

7. a. The area of the iced surfaces of the cake is the sum of the areas of two faces.



Step 1: Calculate the area of A_1. **Note:** The circle has a radius of 10 cm.

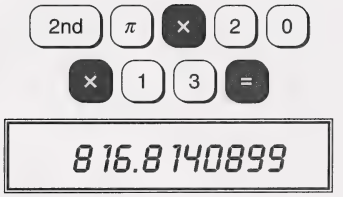
$$A_1 = \pi r^2$$
$$= \pi(10)^2$$
$$\doteq 314.2$$



The area of A_1 is about 314.2 cm^2.

Step 2: Calculate the area of A_2. **Note:** The rectangle has a length of πd, and the width is 13 cm.

$$A_2 = \ell w$$
$$= (\pi d)(13)$$
$$= \pi(20)(13)$$
$$\doteq 816.8$$



The area of A_2 is about 816.8 cm^2.

Step 3: Find the area of the iced surfaces of the cake.

$$\begin{aligned} S &= A_1 + A_2 \\ &\doteq 314.2 + 816.8 \\ &\doteq 1131.0 \end{aligned}$$

The area of the iced surfaces of the cake is about $1131\ \text{cm}^2$.

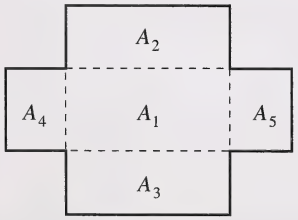
b. Use this reasoning.

The cake's being cut into two layers does not change the overall side area. The original top is also not affected. The only additional area that needs icing is the top surface of the bottom layer. This will have the same area as the original top. So, simply add $314.2\ \text{cm}^2$ to the previous total.

$$1131.0 + 314.2 \doteq 1445.2$$

The area of the iced surfaces of the cake is approximately $1445\ \text{cm}^2$.

8. The surface area is the sum of the areas of five faces. **Note:** $A_2 = A_3$ and $A_4 = A_5$



Step 1: Find the area of the ceiling, A_1. **Note:** The rectangle is $8\ \text{m} \times 4\ \text{m}$.

$$\begin{aligned} A_1 &= \ell w \\ &= 8(4) \\ &= 32 \end{aligned}$$

The area of the ceiling is $32\ \text{m}^2$.

Step 2: Calculate the area of the long walls (A_2 and A_3). **Note:** Each rectangle is $8\ \text{m} \times 3\ \text{m}$.

$$\begin{aligned} A_2 &= \ell w \\ &= 8(3) \\ &= 24 \end{aligned} \qquad \therefore A_3 = 24$$

The area of A_2 and A_3 together is $48\ \text{m}^2$.

Step 3: Calculate the area of the short walls (A_4 and A_5). **Note:** Each rectangle is $4\ \text{m} \times 3\ \text{m}$.

$$\begin{aligned} A_4 &= \ell w \\ &= 4(3) \\ &= 12 \end{aligned} \qquad \therefore A_5 = 12$$

Together the area of A_4 and A_5 is $24\ \text{m}^2$.

Step 4: Calculate the area of the painted surfaces.

$$\begin{aligned} S &= A_1 + (A_2 + A_3) + (A_4 + A_5) \\ &= 32 + 48 + 24 \\ &= 104 \end{aligned}$$

The area of the painted surfaces is $104\ \text{m}^2$.

Step 5: Calculate the amount of paint needed.

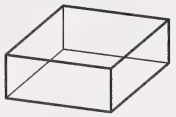
$$104 \div 12 = 8.\overline{6}$$

Because you cannot get a fraction of a pail of paint, 9 pails of paint is needed.

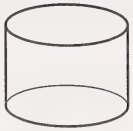
Looking Back

9.

Triangular Prism
The triangular prism is composed of two congruent triangles and three rectangles. To determine the surface area, you need the height of the triangle, the dimensions of the sides of the triangle, and the length of the rectangular face.



Rectangular Prism
The rectangular prism is composed of three pairs of congruent rectangles. To determine the surface area, you need the length and width of the base and the height of the rectangular prism.



Cylinder
The cylinder is composed of two circles and one rectangle. To determine the surface area, you need the height of the cylinder and the radius of the circle.

Section 1: Activity 4

1. a. The volume is 5 cubic units.

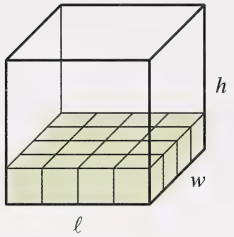
b. The volume is 60 cubic units.

c. The volume is 24 cubic units.

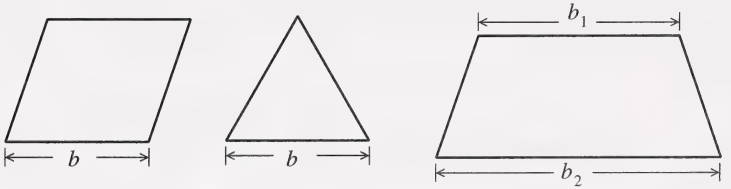
d. The volume is 12 cubic units.

2. The volume is equal to the length times the width times the height.

3. When you use unit cubes to find the volume of a rectangular prism, you count the number of cubes in a single layer and multiply by the number of layers. This is the same as calculating the area of the base $(A = \ell w)$ and multiplying by the height, h.

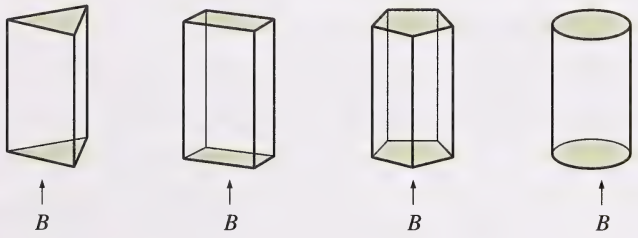


4. a. In these formulas, b represents the length of the bases of the figures.

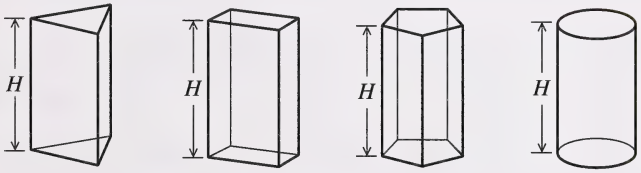


b. In these formulas, h represents the height of the figures.

c. In this formula, B represents the area of the base of the prism or cylinder.



d. In this formula, H represents the height of the prism or cylinder.



5. a. **Step 1:** Calculate B, the area of the base. **Note:** Use one of the triangular ends as the base.

$$\begin{aligned} B &= \frac{1}{2}bh \\ &= \frac{1}{2}(3)(7) \\ &= 10.5 \end{aligned}$$

The area of the base is $10.5\ \text{m}^2$.

Step 2: Calculate V, the volume of the prism.

$$\begin{aligned} V &= BH \\ &= 10.5(12) \\ &= 126 \end{aligned}$$

The volume is $126\ \text{m}^3$.

b. **Step 1:** Calculate B, the area of the base. **Note:** The base is a circle.

$$\begin{aligned} B &= \pi r^2 \\ &= \pi(8)^2 \\ &\doteq 201.06 \end{aligned}$$

The area of the base is about $201.06\ \text{cm}^2$.

Step 2: Calculate V, the volume of the cylinder.

$$\begin{aligned} V &= BH \\ &\doteq 201.06(15) \\ &\doteq 3015.9 \end{aligned}$$

The volume is about $3015.9\ \text{cm}^3$.

Note: Your answer may vary slightly if you multiplied $\pi \times 64 \times 15$.

c. **Step 1:** Calculate B, the area of the base. **Note:** The base is a regular hexagon.

$$\begin{aligned} B &= \frac{nsa}{2} \\ &= \frac{6(0.8)(0.7)}{2} \\ &= 1.68 \end{aligned}$$

Recall:
n = number of sides
s = length of each side
a = length of apothem

The area of the base is $1.68\ \text{m}^2$.

Step 2: Calculate V, the volume of the prism.

$$\begin{aligned} V &= BH \\ &= 1.68(3.2) \\ &\doteq 5.4 \end{aligned}$$

The volume is about $5.4\ \text{m}^3$.

d. **Step 1:** Calculate B, the area of the base. **Note:** The base is a rectangle.

$$\begin{aligned} B &= \ell w \\ &= 20(6) \\ &= 120 \end{aligned}$$

The area of the base is $120\ \text{mm}^2$.

Step 2: Calculate V, the volume of the prism.

$$\begin{aligned} V &= BH \\ &= 120(14) \\ &= 1680 \end{aligned}$$

The volume is $1680\ \text{mm}^3$.

e. **Step 1:** Calculate B, the area of the base. **Note:** The base is a triangle.

$$\begin{aligned} B &= \frac{1}{2}bh \\ &= \frac{1}{2}(12)(8) \\ &= 48 \end{aligned}$$

The area of the base is $48\ \text{cm}^2$.

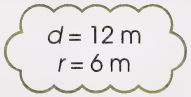
Step 2: Calculate V, the volume of the prism.

$$\begin{aligned} V &= BH \\ &= 48(22) \\ &= 1056 \end{aligned}$$

The volume is $1056\ \text{cm}^3$.

f. Step 1: Calculate B, the area of the base. **Note:** The base is a circle.

$$\begin{aligned} B &= \pi r^2 \\ &= \pi(6)^2 \\ &\doteq 113.10 \end{aligned}$$



The area of the base is about $113.10\ \text{m}^2$.

Step 2: Calculate V, the volume of the cylinder.

$$\begin{aligned} V &= BH \\ &\doteq 113.10(6) \\ &\doteq 678.6 \end{aligned}$$

The volume is about $678.6\ \text{m}^3$.

6. Step 1: Calculate B, the area of the base. **Note:** The base is a regular decagon.

$$\begin{aligned} B &= \frac{nsa}{2} \\ &= \frac{10(5.0)(7.7)}{2} \\ &= 192.5 \end{aligned}$$

The area of the base is $192.5\ \text{cm}^2$.

Step 2: Calculate V, the volume of the prism.

$$\begin{aligned} V &= BH \\ &= 192.5(5.8) \\ &= 1116.5 \end{aligned}$$

The volume is $1116.5\ \text{cm}^3$.

7. Step 1: Calculate B, the area of the base. **Note:** The base is a regular hendecagon (a polygon with 11 sides).

$$\begin{aligned} B &= \frac{nsa}{2} \\ &= \frac{11(8.0)(13.3)}{2} \\ &= 585.2 \end{aligned}$$

The area of the base is $585.2\ \text{mm}^2$.

Step 2: Calculate V, the volume of the prism.

$$\begin{aligned} V &= BH \\ &= 585.2(2.0) \\ &= 1170.4 \end{aligned}$$

The volume is about $1170\ \text{mm}^3$.

8. **Step 1:** Calculate B, the area of the base. **Note:** The base is a circle.

$$\begin{aligned} B &= \pi r^2 \\ &= \pi(5)^2 \\ &\doteq 78.5 \end{aligned}$$

The area of the base is about $78.5\ \text{cm}^2$.

Step 2: Calculate V, the volume of the cylinder.

$$\begin{aligned} V &= BH \\ &\doteq 78.5(12) \\ &\doteq 942 \end{aligned}$$

You would need about $942\ \text{cm}^3$ of wax.

9. **a.** Yes, their heights are the same.

b. Step 4 must be done three times. Thus, the prism holds three times as much as a pyramid having the same height and the same base.

c. $V = \frac{1}{3}BH$

In this formula, B is the area of the base and H is the height of the pyramid.

10. **a.** $B = s^2$

$$\begin{aligned} &= (8)^2 \\ &= 64 \end{aligned}$$

$$\begin{aligned} V &= \frac{1}{3}BH \\ &= \frac{1}{3}(64)(20) \\ &\doteq 426.7 \end{aligned}$$

The volume is about $426.7\ \text{cm}^3$.

b. $B = \frac{1}{2}bh$

$$\begin{aligned} &= \frac{1}{2}(4.8)(7.2) \\ &= 17.28 \end{aligned}$$

$$\begin{aligned} V &= \frac{1}{3}BH \\ &= \frac{1}{3}(17.28)(6.3) \\ &= 36.3 \end{aligned}$$

The volume is about $36.3\ \text{m}^3$.

c. $B = \frac{nsa}{2}$

$= \frac{6(4)(3.5)}{2}$

$= 42$

$V = \frac{1}{3}BH$

$= \frac{1}{3}(42)(7.2)$

$= 100.8$

The volume is $100.8\ \text{cm}^3$.

d. $B = \ell w$

$= (22.4)(13)$

$= 291.2$

$V = \frac{1}{3}BH$

$= \frac{1}{3}(291.2)(28.2)$

$= 2737.3$

The volume of the pyramid is about $2737.3\ \text{m}^3$.

11. a. $B = s^2$

$= (230)^2$

$= 52\,900$

$V = \frac{1}{3}BH$

$= \frac{1}{3}(52\,900)(147)$

$= 2\,592\,100$

The volume of stone in the pyramid is $2\,592\,100\ \text{m}^3$.

b. $2\,592\,100 \div 2\,300\,000 \doteq 1.1$

Each block had an approximate volume of $1.1\ \text{m}^3$.

12. $V = \frac{1}{3}BH$

$= \frac{1}{3}(11.4)(3.2)$

$= 12.16$

The tent holds about $12.2\ \text{m}^3$ of air.

13. Method 1

V of cube $= BH$

$= (s^2)s$

$= s^3$

$= (60)^3$

$= 216\,000$

V of each pyramid $= 216\,000 \div 6$

$= 36\,000$

The volume of each pyramid is $36\,000\ \text{cm}^3$.

Method 2

Find the volume of each pyramid directly. Each pyramid reaches to the centre point of the cube, so the height of the pyramid will be half the width of the cube, or 30 cm.

$B = s^2$

$= (60)^2$

$= 3600$

$V = \frac{1}{3}BH$

$= \frac{1}{3}(3600)(30)$

$= 36\,000$

The volume of each pyramid is $36\,000\ \text{cm}^3$.

14. a. Yes, the radius of the top of the cone is the same as the radius of the glass.

b. The cone must be filled and poured into the glass three times in order to fill the glass completely.

c. You know from the experiment that the volume of the glass is 3 times that of the cone. So, you must divide the volume of the glass by 3 to find the volume of the cone.

d. The formula for volume of a cylinder is $V = BH$. Therefore, the formula for the volume of a cone with the same base and same height is $V = \frac{1}{3}BH$.

15. a.
$$\begin{aligned} V &= \frac{1}{3}BH \\ &= \frac{1}{3}\left(\pi r^2\right)H \\ &= \frac{1}{3}\pi(5)^2(12) \\ &\doteq 314 \end{aligned}$$

The volume is about $314\ \text{cm}^3$.

b.
$$\begin{aligned} V &= \frac{1}{3}BH \\ &= \frac{1}{3}\left(\pi r^2\right)H \\ &= \frac{1}{3}\pi(4.5)^2(9) \\ &\doteq 191 \end{aligned}$$

The volume is about $191\ \text{cm}^3$.

c.
$$\begin{aligned} V &= \frac{1}{3}BH \\ &= \frac{1}{3}\left(\pi r^2\right)H \\ &= \frac{1}{3}\pi(6)^2(14) \\ &\doteq 528 \end{aligned}$$

The volume is about $528\ \text{cm}^3$.

d.
$$\begin{aligned} V &= \frac{1}{3}BH \\ &= \frac{1}{3}\left(\pi r^2\right)H \\ &= \frac{1}{3}\pi(9)^2(20) \\ &\doteq 1696 \end{aligned}$$

The volume is about $1696\ \text{cm}^3$.

16.
$$\begin{aligned} V &= \frac{1}{3}BH \\ &= \frac{1}{3}\left(\pi r^2\right)H \\ &= \frac{1}{3}\pi(4)^2(12) \\ &\doteq 201 \end{aligned}$$

The cone, filled level to its top, can hold approximately $201\ \text{cm}^3$ of crushed ice.

17.
$$\begin{aligned} V &= \frac{1}{3}BH \\ &= \frac{1}{3}\left(\pi r^2\right)H \\ &= \frac{1}{3}\pi(3.5)^2(7.2) \\ &\doteq 92 \end{aligned}$$

Each cup can hold approximately 92 mL $\left(92\ \text{cm}^3\right)$ of water.

18. a.
$$\begin{aligned} V &= BH \\ &= \left(\pi r^2\right)H \\ &= \pi(5)^2(10) \\ &\doteq 785 \end{aligned}$$

The cylinder has a capacity of approximately 785 mL.

b.
$$\begin{aligned} V &= \frac{1}{3}BH \\ &= \frac{1}{3}\left(\pi r^2\right)H \\ &= \frac{1}{3}\pi(5)^2(10) \\ &\doteq 262 \end{aligned}$$

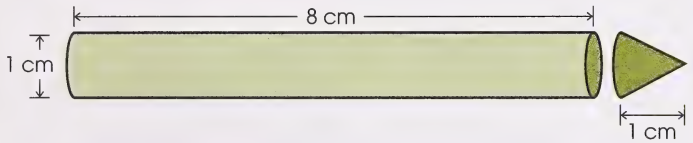
The cone has a capacity of approximately 262 mL.

c. The cylinder fills about $\frac{785}{1000}$ or $\frac{3}{4}$ of the cube. The cone fills about $\frac{262}{100}$ or $\frac{1}{4}$ of the cube.

d. From the previous work, you know the rectangular pyramid would fill $\frac{1}{3}$ of the cube. Therefore, the rectangular pyramid would fill more of the cube than the cone.

Looking Back

19.



Step 1: Find the volume of the cylindrical part of the pencil.

$$\begin{aligned} V &= BH \\ &= \left(\pi r^2\right) H \\ &= \pi (0.5)^2 (8) \\ &\doteq 6.28 \end{aligned}$$

The volume of the cylindrical part is about $6.28\ \text{cm}^3$.

Step 2: Find the volume of the conical part of the pencil.

$$\begin{aligned} V &= \frac{1}{3} BH \\ &= \frac{1}{3}\left(\pi r^2\right) H \\ &= \frac{1}{3}(\pi)(0.5)^2 (1) \\ &\doteq 0.26 \end{aligned}$$

The volume of the conical part is about $0.26\ \text{cm}^3$.

Step 3: Find the total volume of the pencil.

$$6.28 + 0.26 \doteq 6.54$$

The volume of wax needed to make a jumbo crayon is about $6.5\ \text{cm}^3$.

Note: In a multi-step problem, it is helpful to round the answers intermediate steps to one decimal place more than required to avoid cumulative rounding errors.

Section 1: Activity 5

1. $S = 6s^2$

$= 6(4)^2$

$= 96$

$V = \ell wh$

$= 4(4)(4)$

$= 64$

The surface area is $96\ \text{cm}^2$, and the volume is $64\ \text{cm}^3$.

2. a. $S = 6s^2$ $\qquad$ $V = \ell wh$

$= 6(8)^2$ $\qquad$ $= 8(8)(8)$

$= 384$ $\qquad$ $= 512$

The surface area is $384\ \text{cm}^2$, and the volume is $512\ \text{cm}^3$.

b. $\frac{384}{96} = 4$

The surface area increases 4 times.

c. $\frac{512}{64} = 8$

The volume increases 8 times.

3. $4 = 2^2$ and $8 = 2^3$

4. a. $S = 6s^2$ $\qquad$ $V = \ell wh$

$= 6(12)^2$ $\qquad$ $= 12(12)(12)$

$= 864$ $\qquad$ $= 1728$

The surface area is $864\ \text{cm}^2$, and the volume is $1728\ \text{cm}^3$.

b. $\frac{864}{96} = 9$

The surface area increases 9 times.

c. $\frac{1728}{64} = 27$

The volume increases 27 times.

5. $9 = 3^2$ and $27 = 3^3$

6. a.

Change in Dimensions	Change in Surface Area	Change in Volume
doubled	$2^2 \times$	$2^3 \times$
tripled	$3^2 \times$	$3^3 \times$
halved	$\left(\frac{1}{2}\right)^2 \times$	$\left(\frac{1}{2}\right)^3 \times$
n times	$n^2 \times$	$n^3 \times$

b. Volume changes faster.

c. Surface area changes by a factor of the square of the change in dimensions. Volume changes by a factor of the cube of the change in dimensions.

d. The exponent for change in surface area is 2 because surface area involves a product of 2 dimensions $(\ell \times w)$. Likewise, the exponent for change in volume is 3 because volume involves a product of 3 dimensions $(\ell \times w \times h)$.

7. **Step 1:** Calculate the area of the top and bottom faces.

$$\begin{aligned} A &= \ell w \\ &= 30(8) \\ &= 240 \end{aligned}$$

The area of each face is $240\ \text{cm}^2$.

Together, the top and bottom faces have an area of $2 \times 240 = 480\ \text{cm}^2$.

Step 2: Calculate the area of the front and back faces.

$$\begin{aligned} A &= \ell w \\ &= 30(20) \\ &= 600 \end{aligned}$$

The area of each face is $600\ \text{cm}^2$.

Together, the front and back faces have an area of $2 \times 600 = 1200\ \text{cm}^2$.

Step 3: Calculate the area of the left and right faces.

$$\begin{aligned} A &= \ell w \\ &= 20(8) \\ &= 160 \end{aligned}$$

The area of each face is $160\ \text{cm}^2$.

Together, the left and right faces have an area of $2 \times 160 = 320\ \text{cm}^2$.

Step 4: Calculate the surface area of the prism. It is the total sum of the areas of top and bottom, front and back, and left and right faces.

$$\begin{aligned} S &= 480 + 1200 + 320 \\ &= 2000 \end{aligned}$$

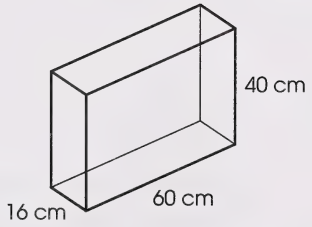
The surface area of the container is $2000\ \text{cm}^2$.

Step 5: Find the volume of the container.

$$\begin{aligned} V &= \ell wh \\ &= 30(8)(20) \\ &= 4800 \end{aligned}$$

The volume of the prism is $4800\ \text{cm}^3$.

8. **a.** **Step 1:** Calculate the area of the top and bottom faces.



$$\begin{aligned} A &= \ell w \\ &= 60 \times 16 \\ &= 960 \end{aligned}$$

The area of each face is $960\ \text{cm}^2$.

Together, the top and bottom faces have an area of $2 \times 960 = 1920\ \text{cm}^2$.

Step 2: Calculate the area of the front and back faces.

$$\begin{aligned} A &= \ell w \\ &= 60 \times 40 \\ &= 2400 \end{aligned}$$

The area of each face is $2400\ \text{cm}^2$.

Together, the front and back faces have an area of $2 \times 2400 = 4800\ \text{cm}^2$.

Step 3: Calculate the area of the left and right faces.

$$\begin{aligned} A &= \ell w \\ &= 40 \times 16 \\ &= 640 \end{aligned}$$

The area of each face is $640\ \text{cm}^2$.

Together, the left and right faces have an area of $2 \times 640 = 1280\ \text{cm}^2$.

Step 4: Calculate the surface area of the prism. It is the total of the areas of top and bottom, front and back, and left and right faces.

$$\begin{aligned} S &= 1920 + 4800 + 1280 \\ &= 8000 \end{aligned}$$

The surface area of the prism is $8000\ \text{cm}^2$.

Step 5: Find the volume of the prism.

$$\begin{aligned} V &= \ell wh \\ &= 60(16)(40) \\ &= 38\ 400 \end{aligned}$$

The volume of the prism is $38\ 400\ \text{cm}^3$.

b. Yes. Based on the calculations, doubling the dimensions causes the surface area to increase by a factor of 2^2 whether the object is a cube or a rectangular prism. Also, doubling the dimensions causes the volume to increase by a factor of 2^3 whether the object is a cube or a rectangular prism. The relationship that holds for a cube holds for a rectangular prism.

Doubling the dimensions of this rectangular prism had the following effect on the surface area and volume:

- The surface area increased by a factor of $\frac{8000}{2000} = 4$.
- The surface area increased 2^2 times.
- The volume increased by a factor of $\frac{38\ 400}{4800} = 8$.
- The volume increased by 2^3 times.

9. a. It takes four times more material to make the box, and it holds eight times more cereal.

b. It's cheaper for the manufacturer to package cereal in larger boxes. That's why buying larger quantities is often cheaper for the consumer.

c. Cereal packages come in a variety of sizes to serve the needs and desires of a variety of consumers (such as single people, small families, large families, and frequent shoppers).

10. a. The ratio is $3:1$ $\left(\frac{18}{6}, \frac{30}{10}, \text{ and } \frac{75}{25}\right)$.

b. Three times more reinforcing tape is needed for the larger box $(164 \text{ cm} \rightarrow 492 \text{ cm})$.

c. Nine times $\left(3^2\right)$ more material is needed to build the larger box $\left(920 \text{ cm}^2 \rightarrow 8280 \text{ cm}^2\right)$.

d. The volume of the larger box is 27 times $\left(3^3\right)$ the volume of the smaller box $\left(1500 \text{ cm}^3 \rightarrow 40\,500 \text{ cm}^3\right)$.

11. a. Based on previous calculations for surface area and volume, the surface area should be four times larger $\left(\text{or } 1608 \text{ cm}^2\right)$ and the volume should be eight times larger $\left(\text{or } 4824 \text{ cm}^3\right)$.

b. Step 1: Calculate the area of the top and bottom faces of the can.

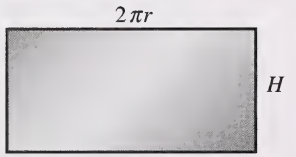
$$\begin{aligned} A &= \pi r^2 \\ &= \pi(8)^2 \\ &\doteq 201.1 \end{aligned}$$

The area of each face is approximately 201.1 cm^2.

Together, the top and bottom faces have an area of $2 \times 201.1 \doteq 402.2 \text{ cm}^2$.

Step 2: Calculate the area of the curved surface of the can.

$$\begin{aligned} A &= \ell w \\ &= (2\pi r) H \\ &= 2\pi(8)(24) \\ &\doteq 1206.4 \end{aligned}$$



The area of the curved surface is approximately 1206.4 cm^2.

Step 3: Calculate the surface area of the soup can.

$$\begin{aligned} S &\doteq 402.2 + 1206.4 \\ &\doteq 1608.6 \end{aligned}$$

The surface area of the soup can is approximately 1609 cm^2.

Step 4: Calculate the volume of the soup can.

$$B = \pi r^2 \qquad V = BH$$
$$= \pi\left(8^2\right) \qquad \doteq (201.1)(24)$$
$$\doteq 201.1 \qquad \doteq 4826.4$$

The volume of the soup can is approximately $4826\ \text{cm}^3$.

Step 5: Determine the factors by which the surface area and volume increase as a consequence of doubling the dimensions of the soup can.

$$\frac{\text{new surface area}}{\text{original surface area}} \doteq \frac{1609}{402}$$
$$\doteq 4.0$$
$$\doteq 2^2$$

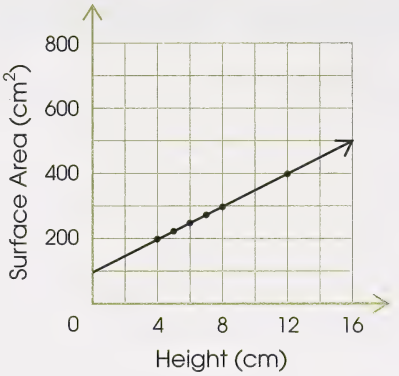
$$\frac{\text{new volume}}{\text{original volume}} \doteq \frac{4826}{603}$$
$$\doteq 8.0$$
$$\doteq 2^3$$

The surface area is four times (or 2^2 times) larger, and the volume is eight times (or 2^3 times) larger.

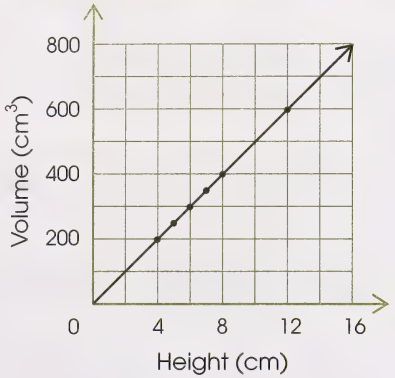
12. a. The following chart shows the relationship among height, surface area, and volume of a can with a radius of 4 cm.

Height, h (cm)	Surface Area, S $\left(\text{cm}^2\right)$	Volume, V $\left(\text{cm}^3\right)$
4	201	201
5	226	251
6	251	302
7	276	352
8	302	402
12	402	603

b. Surface Area Related to Height



Volume Related to Height



Volume changes more quickly than surface area when the height changes.

c. The surface area of a can with a height of 0 cm is approximately $101\ \text{cm}^2$. This is the surface area of the two ends of the can.

The volume of a can with a height of 0 cm is $0\ \text{cm}^3$. If there is no height, then there is no volume. You need three dimensions for volume.

13. a. **Step 1:** Calculate the areas of the top and bottom faces.

$$\begin{aligned} A &= \pi r^2 \\ &= \pi (10)^2 \\ &\doteq 314.2 \end{aligned}$$

The area of each face is approximately $314.2\ \text{cm}^2$.

Together, the top and bottom faces have an area of $2 \times 314.2 \doteq 628.4\ \text{cm}^2$.

Step 2: Calculate the area of the curved surface.

$$\begin{aligned} A &= \ell w \\ &= (2\pi r) H \\ &= 2\pi (10) 20 \\ &\doteq 1256.6 \end{aligned}$$

The area of the curved surface is approximately $1256.6\ \text{cm}^2$.

Step 3: Calculate the surface area of the cylindrical container.

$$\begin{aligned} S &\doteq 314.2 + 314.2 + 1256.6 \\ &\doteq 1885.0 \end{aligned}$$

The surface area of the cylindrical container is about $1885\ \text{cm}^2$.

b.
$$\begin{aligned} S &= 6s^2 \\ &= 6(18.45)^2 \\ &\doteq 2042.4 \end{aligned}$$

The surface area of the cube is approximately $2042\ \text{cm}^2$.

c. The surface area of the cylinder is less than the surface area of the cube.

d. The cylinder uses less material to hold the same volume. Therefore, the packaging industry will try to use cylindrical containers where possible in order to save costs.

Now Try This

14. a. The first four diagrams in the progression had these bottom layers:

1, 2 by 2, 3 by 3, and 4 by 4

So, the next layer will be 25. Thus, the next pyramidal number is $30 + 25 = 55$.

b. The numbers that make up each row are called square numbers.

c. You would add the first ten square numbers.

Looking Back

15. a.

Volume (cm^3)	Length (cm)	Width (cm)	Height (cm)	Surface Area (cm^2)
4000	10	10	40	1700
4000	15	15	17.778	1291.667
4000	20	20	10	1200
4000	19	19	11.080	1203.105
4000	21	21	9.070	1202.905

Note: The heights are determined by dividing the volume by the area of the base.

The most efficient size for a square-bottomed container holding $4000\ cm^3$ would be $20\ cm \times 20\ cm \times 10\ cm$.

b. In an earlier question, it was concluded that a cylinder has less surface area for a given volume than a cube. Therefore, a container with a circle as its base may be more efficient.

The following trial-and-error calculations confirm that the most efficient cylinder-shaped container has a smaller surface area for a volume of $4000\ cm^3$ than a cube of the same volume.

Volume (cm^3)	Radius (cm)	Height (cm)	Surface Area (cm^2)
4000	5	50.930	1678.540
4000	10	12.732	1114.159
4000	15	5.659	1240.192
4000	9	15.719	1143.358
4000	11	10.523	1107.405

Note: The heights are determined by dividing the volume by the area of the base.

c. Other factors that may influence the size and shape of a container are as follows:

- ease of handling
- ease of storage before use

Section 2: Activity 1

1. a. $5+x=13$ **b.** $5+4=x$

c. $8-3=x+4$ **d.** $4+1=x-2$

2. a. $5+\mathbf{8}=13$ **b.** $5+4=\mathbf{9}$

c. $8-3=\mathbf{1}+4$ **d.** $4+1=\mathbf{7}-2$

3. a. Subtract 2 or add -2 to each side.

b. Add 5 to each side.

c. Subtract 3 or add -3 to each side.

d. Add 2 to each side.

e. Divide each side by 3 or multiply each sidy by $\frac{1}{3}$.

f. Divide each side by -2 or multiply each side by $-\frac{1}{2}$.

g. Multiply each side by 4.

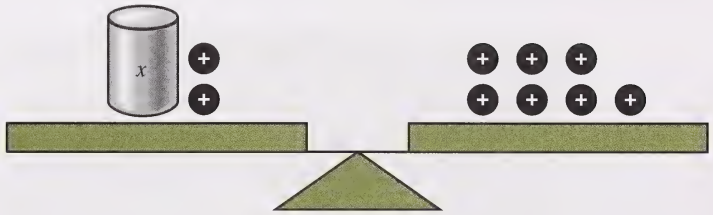
h. Multiply each side by 2.

4. a.
$$x+2=7$$
$$x+2-2=7-2$$
$$x=5$$

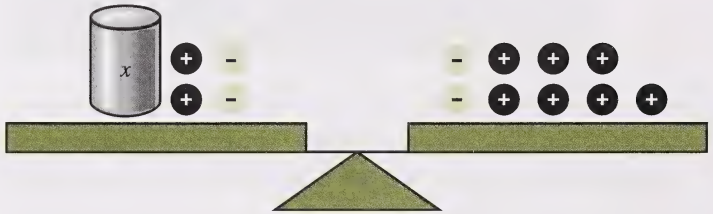
$$\begin{array}{r} x+2= \ 7 \\ -2 \ -2 \\ \hline x \ \ = 5 \end{array}$$

Following is an explanation of how learning aids can be used to illustrate the answer.

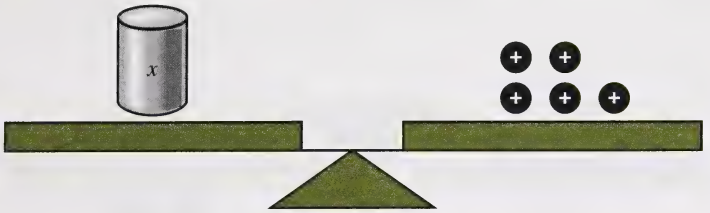
Step 1: Model $x+2=7$.



Step 2: Isolate the cylinder. Add two negative counters to **each** side of the equation.



Step 3: Simplify by removing any zero pairs. This is the result.



The model illustrates that $x = 5$.

Check

LS	RS
$x+2$	7
$=5+2$	
$=7$	

LS = RS

b. $t-5=1$

$t-5+5=1+5$

$t=6$

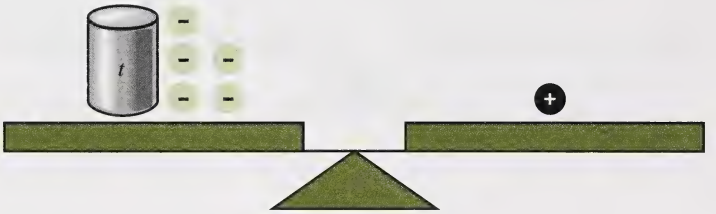
$t-5=1$

$+5 \quad +5$

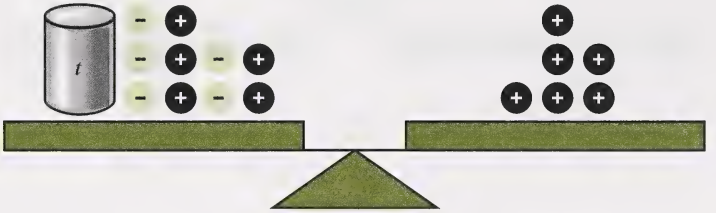
$t = 6$

Following is an explanation of how learning aids can be used to illustrate the answer.

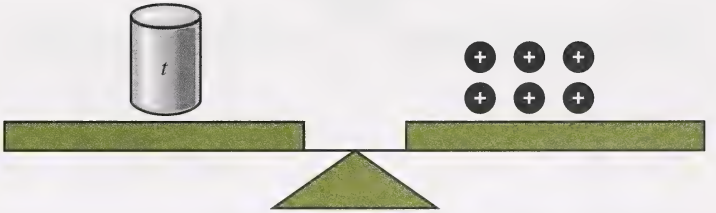
Step 1: Model $t-5=1$.



Step 2: Isolate the cylinder. Add five positive counters to **each** side of the equation.



Step 3: Simplify by removing any zero pairs. This is the result.



The model illustrates that $t = +6$.

Check

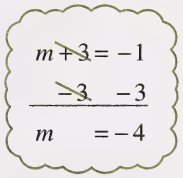
LS	RS
$t-5$	1
$=6-5$	
$=1$	

LS = RS

c. $m+3=-1$

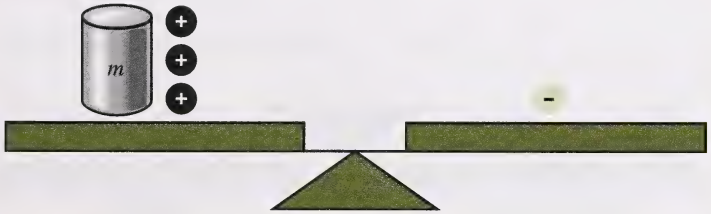
$m+3-3=-1-3$

$m=-4$

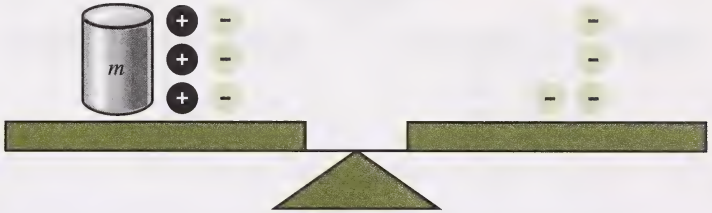


Following is an explanation of how learning aids can be used to illustrate the answer.

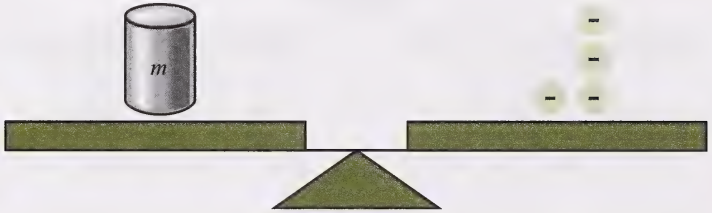
Step 1: Model $m+3=-1$.



Step 2: Isolate the cylinder. Add three negative counters to **each** side.



Step 3: Simplify by removing any zero pairs. This is the result.



The model illustrates that $m=-4$.

Check

LS	RS
$m+3$	-1
$=-4+3$	
$=-1$	

LS = RS

d. $y-2=-8$

$y-2+2=-8+2$

$y=-6$

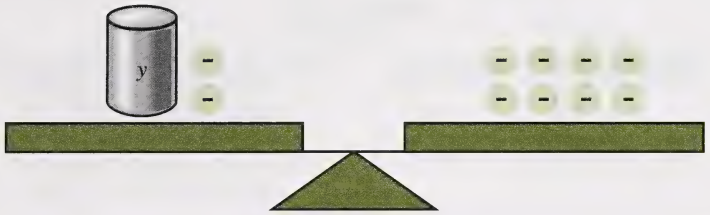
$y-2=-8$

$+2 \quad +2$

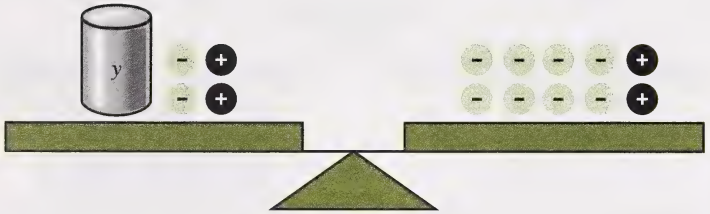
$y \quad =-6$

Following is an explanation of how learning aids can be used to illustrate the answer.

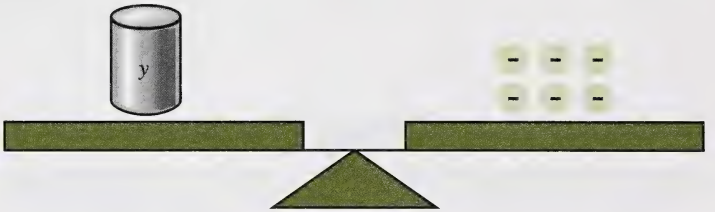
Step 1: Model $y-2=-8$.



Step 2: Isolate the cylinder. Add two positive counters to **each** side.



Step 3: Simplify by removing any zero pairs. This is the result.



The model illustrates that $y=-6$.

Check

LS	RS
$y-2$	-8
$=-6-2$	
$=-8$	

LS = RS

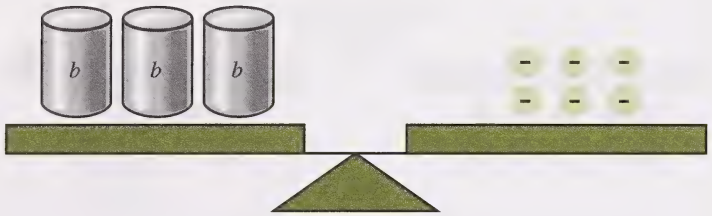
e. $3b=-6$

$\frac{3b}{3}=\frac{-6}{3}$

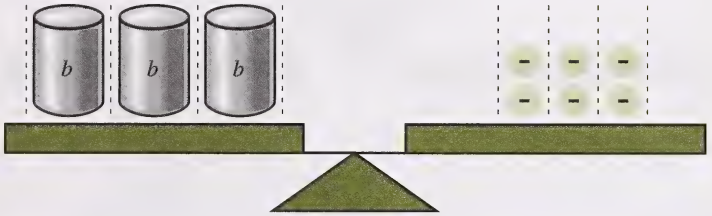
$b=-2$

Following is an explanation of how learning aids can be used to illustrate the answer.

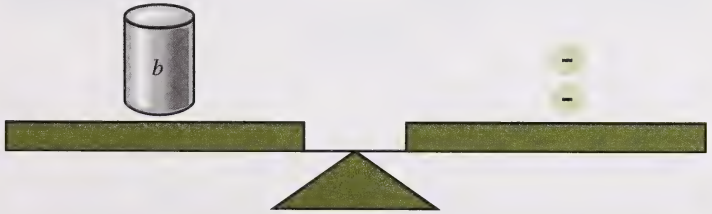
Step 1: Model $3b = -6$.



Step 2: Arrange **each** side into three equal groups.



Step 3: Keep **one** group from **each** side of the scale. This is the same as dividing each side by 3.



The model illustrates that $b = -2$.

Check

LS	RS
$3b$	-6
$=3(-2)$	
$=-6$	

LS = RS

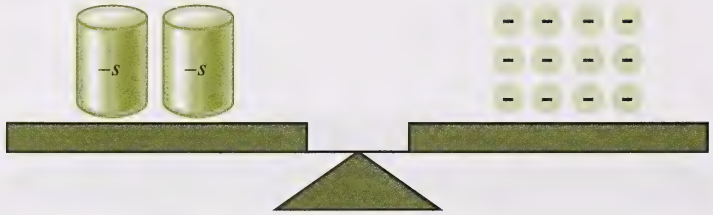
f. $-2s = -12$

$$\frac{-2s}{-2} = \frac{-12}{-2}$$

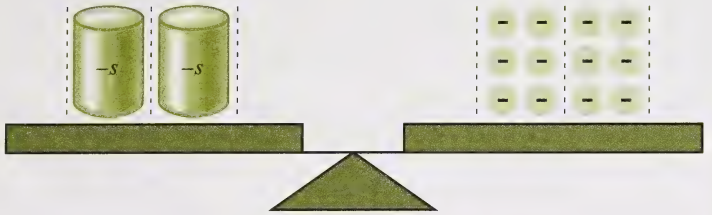
$$s = +6$$

Following is an explanation of how learning aids can be used to illustrate the answer.

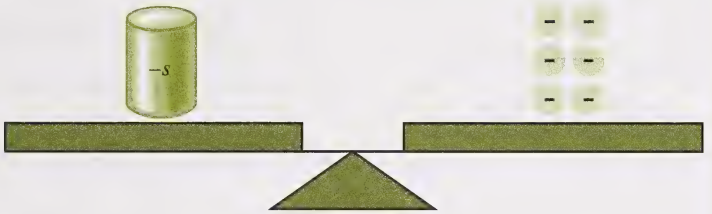
Step 1: Model $-2s = -12$.



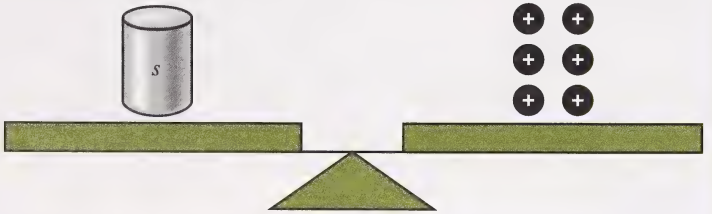
Step 2: Arrange **each** side into two equal groups.



Step 3: Keep **one** group from **each** side of the scale.



Step 4: To solve for s, trade the learning aids on each side for their opposites. **Note:** The opposite of $-s$ is s, and the opposite of -6 is $+6$ or 6.



The model illustrates that $s = 6$.

Check

LS	RS
$-2s$	-12
$=-2(6)$	
$=-12$	

LS = RS

g. $\frac{n}{4} = 3$

$4 \times \left(\frac{n}{4}\right) = 4 \times (3)$

$n = 12$

Check

LS	RS
$\frac{n}{4}$	3
$=\frac{12}{4}$	
$=3$	

LS = RS

h. $\frac{m}{2} = -6$

$2 \times \left(\frac{m}{2}\right) = 2 \times (-6)$

$m = -12$

Check

LS	RS
$\frac{m}{2}$	-6
$=\frac{-12}{2}$	
$=-6$	

LS = RS

5. **Step 1:** Write an equation to describe the situation. Let n be the number of tea bags in a box.

$$n + 5 = 125$$

Step 2: Solve the equation.

$$n + 5 - 5 = 125 - 5$$
$$n = 120$$

The box had 120 tea bags.

Check

Determine if the answer is reasonable.

Does 5 tea bags plus 120 tea bags equal 125 tea bags?

$$5 + 120 = 125$$

Yes, the answer is reasonable.

6. **Step 1:** Write an equation to describe the situation. Let n be the number of days in the term.

$$6n = 654$$

Step 2: Solve the equation.

$$\frac{6n}{6} = \frac{654}{6}$$
$$n = 109$$

The term had 109 days.

Check

Determine if the answer is reasonable.

Does 6 sheets of paper per day for 109 days equal 654 sheets of paper?

$$109 \times 6 = 654$$

Yes, the answer is reasonable.

7. **Step 1:** Write an equation to describe the situation. Let p be the price (in dollars) of the adult ticket.

$$p + 5 = 23$$

Step 2: Solve the equation.

$$p + 5 - 5 = 23 - 5$$
$$p = 18$$

The price of the adult ticket was \$18.

Check

Determine if the answer is reasonable.

Does a child's ticket of \$5 and an adult's ticket of \$18 cost \$23?

$$5 + 18 = 23$$

Yes, the answer is reasonable.

8. **Step 1:** Write an equation to describe the situation. Let ℓ be the length (in centimetres) of the card.

$$\ell - 8 = 12$$

Step 2: Solve the equation.

$$\ell - 8 + 8 = 12 + 8$$
$$\ell = 20$$

The length is 20 cm.

Check

Determine if the answer is reasonable.

Is 20 cm minus 12 cm equal to 8 cm?

$$20 - 12 = 8$$

Yes, the answer is reasonable.

9. **Step 1:** Write an equation to describe the situation. Let ℓ be the length (in metres) of the fabric.

$$\frac{\ell}{6} = 2.75$$

Step 2: Solve the equation.

$$6\left(\frac{\ell}{6}\right) = 6(2.75)$$
$$\ell = 16.5$$

The original length of fabric was 16.5 m.

Check

Determine if the answer is reasonable.

If 16.5 m is divided by 6, is each piece 2.75 m?

$$\frac{16.5}{6} = 2.75$$

Yes, the answer is reasonable.

10. **Step 1:** Write an equation to describe the situation. Let r be the regular price (in dollars).

$$\frac{3r}{4} = 6$$

Step 2: Clear the denominator by multiplying each side by 4.

$$4\left(\frac{3r}{4}\right) = 4(6)$$
$$3r = 24$$

Step 3: Solve for r and answer the question.

$$\frac{3r}{3} = \frac{24}{3}$$
$$r = 8$$

The regular price is \$8.

Check

Check the answer.

Is $\frac{3}{4}$ of 8 equal to 6?

$$\frac{3}{\cancel{4}_{1}} \times \cancel{8}^{2} = 6$$

Yes, the answer is reasonable.

11. **Step 1:** Write an equation to describe the situation. Let r be the regular price (in dollars).

$$\frac{2r}{3} = 12$$

Step 2: Clear the denominator by multiplying each side by $\frac{3}{2}$.

$$\frac{\cancel{3}^{1}}{\cancel{2}_{1}}\left(\frac{\cancel{2}r}{\cancel{3}_{1}}\right) = \frac{3}{\cancel{2}_{1}}\left(\cancel{12}^{6}\right)$$
$$r = 18$$

The regular price is \$18.

Check

Suppose the regular price of the sweater is \$18. If the sweater was on sale for one-third off, would the sale price be \$12?

$$\frac{1}{3} \times 18 = 6$$

$$18 - 6 = 12$$

Yes, the answer is reasonable.

12. **a.**
$$3a - 2 = 7$$
$$3a - 2 + 2 = 7 + 2$$
$$3a = 9$$
$$\frac{3a}{3} = \frac{9}{3}$$
$$a = 3$$

Check

LS	RS
$3a - 2$	7
$= 3(3) - 2$	
$= 9 - 2$	
$= 7$	

LS = RS

b.
$$2b + 1 = -9$$
$$2b + 1 - 1 = -9 - 1$$
$$2b = -10$$
$$\frac{2b}{2} = \frac{-10}{2}$$
$$b = -5$$

Check

LS	RS
$2b + 1$	-9
$= 2(-5) + 1$	
$= -10 + 1$	
$= -9$	

LS = RS

c. $\frac{c}{3}-4=5$

$\frac{c}{3}-4+4=5+4$

$\frac{c}{3}=9$

$3\left(\frac{c}{3}\right)=3(9)$

$c=27$

Check

LS	RS
$\frac{c}{3}-4$	5
$=27-4$	
$=9-4$	
$=5$	

LS = RS

d. $\frac{d}{2}+1=5$

$\frac{d}{2}+1-1=5-1$

$\frac{d}{2}=4$

$2\left(\frac{d}{2}\right)=2(4)$

$d=8$

Check

LS	RS
$\frac{d}{2}+1$	5
$=\frac{8}{2}+1$	
$=4+1$	
$=5$	

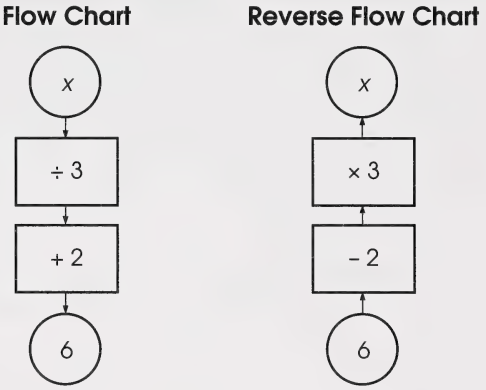
LS = RS

Looking Back

13. a. The variable x is divided by 3 and then 2 is added which equals 6.

To undo the addition operation, you need to subtract 2 from both sides. To undo the division, you need to multiply both sides by 3.

b. Subtraction will have to be done before multiplication. A reverse flow chart explains why subtraction is done first.



Section 2: Activity 2

1. a. The opposite of $+4$ is -4. By adding the opposite, it leaves $3m$ by itself on the left side of the equation.

b. The opposite of m is $-m$. By adding the opposite, it removes the m from the right side of the equation and collects them all on the left side.

c. Both sides of the equation were divided by 2 in order to isolate the variable (remove the 2 from in front of the variable).

2. a.

$$5n-8=3n$$
$$5n-8+8=3n+8$$
$$5n=3n+8$$
$$5n-3n=3n+8-3n$$
$$2n=8$$
$$\frac{2n}{2}=\frac{8}{2}$$
$$n=4$$

b.

$$10-d=-6d$$
$$10-d-10=-6d-10$$
$$-d=-6d-10$$
$$-d+6d=-6d-10+6d$$
$$5d=-10$$
$$\frac{5d}{2}=\frac{-10}{2}$$
$$d=-2$$

c.

$$4n+12=-2n$$
$$4n+12-12=-2n-12$$
$$4n=-2n-12$$
$$4n+2n=-2n-12+2n$$
$$6n=-12$$
$$\frac{6n}{6}=\frac{-12}{6}$$
$$n=-2$$

d.

$$3n+48=5n$$
$$3n+48-48=5n-48$$
$$3n=5n-48$$
$$3n-5n=5n-48-5n$$
$$-2n=-48$$
$$\frac{-2n}{-2}=\frac{-48}{-2}$$
$$n=24$$

e.

$$3.2a-14=1.2a$$
$$3.2a-14+14=1.2a+14$$
$$3.2a=1.2a+14$$
$$3.2a-1.2a=1.2a+14-1.2a$$
$$2a=14$$
$$\frac{2a}{2}=\frac{14}{2}$$
$$a=7$$

f.

$$1.6y=2.8y-6$$
$$1.6y-2.8y=2.8y-6-2.8y$$
$$-1.2y=-6$$
$$\frac{-1.2y}{-1.2}=\frac{-6}{-1.2}$$
$$y=5$$

3. a.

$$3x+9=2x-5$$
$$3x+9-9=2x-5-9$$
$$3x=2x-14$$
$$3x-2x=2x-14-2x$$
$$x=-14$$

b.

$$4-8a=2a-66$$
$$4-8a-4=2a-66-4$$
$$-8a=2a-70$$
$$-8a-2a=2a-70-2a$$
$$-10a=-70$$
$$\frac{-10a}{-10}=\frac{-70}{-10}$$
$$a=7$$

c.

$$3+2a=-5a-11$$
$$3+2a-3=-5a-11-3$$
$$2a=-5a-14$$
$$2a+5a=-5a-14+5a$$
$$7a=-14$$
$$\frac{7a}{7}=\frac{-14}{7}$$
$$a=-2$$

d.

$$8n-2=6n+8$$
$$8n-2+2=6n+8+2$$
$$8n=6n+10$$
$$8n-6n=6n+10-6n$$
$$2n=10$$
$$\frac{2n}{2}=\frac{10}{2}$$
$$n=5$$

4. The following steps describe how to model the solution to the equation $3x - 2 = x$.

Step 1: Model the equation by placing 3 positive cylinders and 2 negative counters on the left side of the balance and 1 positive cylinder on the right side.

Step 2: To isolate $3x$ on the left, place 2 positive counters on each side of the balance.

Step 3: Simplify by removing the zeros from the left side.

Step 4: To isolate the constant, 2, on the right, place a negative cylinder on each side.

Step 5: Simplify by removing the zeros from each side.

Step 6: To isolate x, divide each side into two groups. Keep one of the groups from each side of the balance. The solution is the value of one group.

5. a.

$$\begin{aligned} 2.6c - 2.5 &= 1.2c + 5.2 \\ 2.6c - 2.5 + 2.5 &= 1.2c + 5.2 + 2.5 \\ 2.6c &= 1.2c + 7.7 \\ 2.6c - 1.2c &= 1.2c + 7.7 - 1.2c \\ 1.4c &= 7.7 \\ \frac{1.4c}{1.4} &= \frac{7.7}{1.4} \\ c &= 5.5 \end{aligned}$$

b.

$$\begin{aligned} 1.1 + 3.2s &= 5s - 11.86 \\ 1.1 + 3.2s - 1.1 &= 5s - 11.86 - 1.1 \\ 3.2s &= 5s - 12.96 \\ 3.2s - 5s &= 5s - 12.96 - 5s \\ -1.8s &= -12.96 \\ \frac{-1.8s}{-1.8} &= \frac{-12.96}{-1.8} \\ s &= 7.2 \end{aligned}$$

6. Let n be Anton's hourly wage. Therefore, $3n$ is Mr. Mironov's hourly wage. Write the equation.

$$\begin{aligned} n + 11.60 &= 3n \\ n + 11.60 - 11.60 &= 3n - 11.60 \\ n &= 3n - 11.60 \\ n - 3n &= 3n - 11.60 - 3n \\ -2n &= -11.60 \\ \frac{-2n}{-2} &= \frac{-11.60}{-2} \\ n &= 5.80 \end{aligned}$$

Now, find Mr. Mironov's hourly wage.

$$\begin{aligned} 3n &= 3 \times 5.80 \\ &= 17.40 \end{aligned}$$

Anton makes \$5.80/h, and Mr. Mironov makes \$17.40/h.

7. Since the cost of each pizza is to be the same, set the two equations equal to each other and solve for t.

$$\begin{aligned}
5.90 + 0.90t &= 7.25 + 0.45t \\
5.90 + 0.90t - 5.90 &= 7.25 + 0.45t - 5.90 \\
0.90t &= 1.35 + 0.45t \\
0.90t - 0.45t &= 1.35 + 0.45t - 0.45t \\
0.45t &= 1.35 \\
\frac{0.45t}{0.45} &= \frac{1.35}{0.45} \\
t &= 3
\end{aligned}$$

The cost is the same when 3 toppings are ordered.

8. Answers will vary. A sample scenario is given.

Lisa works at Wholesale Warehouse Ltd. Lisa's supervisor makes 4 times what she makes. If Lisa made \$17.40/h more, she would make the same wage as her supervisor. What is Lisa's hourly wage and her supervisor's hourly wage?

9. a.

$$\begin{aligned}
5(3x - 8) &= 2(2x + 13) \\
15x - 40 &= 4x + 26 \\
15x - 40 + 40 &= 4x + 26 + 40 \\
15x &= 4x + 66 \\
15x - 4x &= 4x + 66 - 4x \\
11x &= 66 \\
\frac{11x}{11} &= \frac{66}{11} \\
x &= 6
\end{aligned}$$

Check

LS	RS
$5(3x-8)$	$2(2x+13)$
$=5[3(6)-8]$	$=2[2(6)+13]$
$=5(18-8)$	$=2(12+13)$
$=5(10)$	$=2(25)$
$=50$	$=50$

LS = RS

Therefore, the solution is $x = 6$.

b.

$$\begin{aligned}
4(a+6) &= 8(a+4) \\
4a+24 &= 8a+32 \\
4a+24-24 &= 8a+32-24 \\
4a &= 8a+8 \\
4a-8a &= 8a+8-8a \\
-4a &= 8 \\
\frac{-4a}{-4} &= \frac{8}{-4} \\
a &= -2
\end{aligned}$$

Check

LS	RS
$4(a+6)$	$8(a+4)$
$=4(-2+6)$	$=8(-2+4)$
$=4(4)$	$=8(2)$
$=16$	$=16$

LS = RS

Therefore, the solution is $a=-2$.

c.

$$\begin{aligned}
-4(2x-3) &= 2(-x+3) \\
-8x+12 &= -2x+6 \\
-8x+12-12 &= -2x+6-12 \\
-8x &= -2x-6 \\
-8x+2x &= -2x-6+2x \\
-6x &= -6 \\
\frac{-6x}{-6} &= \frac{-6}{-6} \\
x &= 1
\end{aligned}$$

Check

LS	RS
$-4(2x-3)$	$2(-x+3)$
$=-4[2(1)-3]$	$=2[-(1)+3]$
$=-4(2-3)$	$=2(-1+3)$
$=-4(-1)$	$=2(2)$
$=4$	$=4$

LS = RS

Therefore, the solution is $x=1$.

d.

$$0.6(2h+6)=1.4(h+2)$$
$$1.2h+3.6=1.4h+2.8$$
$$1.2h+3.6-3.6=1.4h+2.8-3.6$$
$$1.2h=1.4h-0.8$$
$$1.2h-1.4h=1.4h-0.8-1.4h$$
$$-0.2h=-0.8$$
$$\frac{-0.2h}{-0.2}=\frac{-0.8}{-0.2}$$
$$h=4$$

Check

LS	RS
$0.6(2h+6)$	$1.4(h+2)$
$=0.6[2(4)+6]$	$=1.4(4+2)$
$=0.6(8+6)$	$=1.4(6)$
$=0.6(14)$	$=8.4$
$=8.4$	

LS = RS

Therefore, the solution is $h=4$.

10.

Person	Present Age	Age Two Years Ago
Vladimir	$2(n+4)$	$2(n+4)-2$
Enzo	n	$n-2$

Write and solve the equation.

$$2(n+4)-2=3(n-2)$$
$$2n+8-2=3n-6$$
$$2n+6=3n-6$$
$$2n+6-6=3n-6-6$$
$$2n=3n-12$$
$$2n-3n=3n-12-3n$$
$$-n=-12$$
$$n=12$$

Now, find Vladimir's present age.

$$2(n+4)=2(12+4)$$
$$=2(16)$$
$$=32$$

Enzo is presently 12 years old, and Vladimir is presently 32 years old.

11. a. Let w be Mrs. Wasylyk's investment. Thus, $5w$ is Mrs. Chiu's investment, and $3(w-20)$ is Mrs. Nepoose's investment.

Write and solve the equation.

$$\begin{aligned}
5w-3(w-20) &= w+1000\\
5w-3w+60 &= w+1000\\
2w+60 &= w+1000\\
2w+60-60 &= w+1000-60\\
2w &= w+940\\
2w-w &= w+940-w\\
w &= 940
\end{aligned}$$

Now, determine Mrs. Chiu's and Mrs. Nepoose's investment.

$$\begin{aligned}
5w &= 5(940) & 3(w-20) &= 3(940-20)\\
&= 4700 & &= 3(920)\\
& & &= 2760
\end{aligned}$$

Therefore, Mrs. Wasylyk's initial investment was \$940, Mrs. Chiu's was \$4700, and Mrs. Nepoose's was \$2760.

b. $940 \div 400 = 2.35$

Therefore, each share cost \$2.35.

c. $$\begin{aligned}
\frac{1300-940}{400} &= \frac{360}{400}\\
&= 0.9
\end{aligned}$$

Each share has increased in price by \$0.90.

d. $$\begin{aligned}
\text{Percent increase} &= \frac{0.90}{2.35}\times 100\%\\
&\doteq 0.3830\times 100\%\\
&\doteq 38\%
\end{aligned}$$

The percent increase in the value of Mrs. Wasylyk's shares is approximately 38%.

12.

Type of Coin	Number of Coins	Value of Coin (¢)	Total Value (¢)
nickel	n	5	$5n$
dime	$2n$	10	$10(2n)=20n$
quarter	$4(2n)=8n$	25	$25(8n)=200n$
loonie	$8n+10$	100	$100(8n+10)$ $=800n+1000$

Write and solve the equation.

$$\begin{aligned} 5n+20n+200n+800n+1000 &= 13\,300 \\ 1025n+1000 &= 13\,300 \\ 1025n+1000-1000 &= 13\,300-1000 \\ 1025n &= 12\,300 \\ \frac{1025n}{1025} &= \frac{12\,300}{1025} \\ n &= 12 \end{aligned}$$

You know n must be a whole number.

Now, determine the number of dimes, quarters, and loonies individually.

$$\begin{aligned} 2n &= 2(12) \\ &= 24 \end{aligned} \qquad \begin{aligned} 8n &= 8(12) \\ &= 96 \end{aligned} \qquad \begin{aligned} 8n+10 &= 8(12)+10 \\ &= 96+10 \\ &= 106 \end{aligned}$$

Therefore, there were 12 nickels, 24 dimes, 96 quarters, and 106 loonies in the vending machine.

13. Note that some of the steps may be completed in different orders.

a.

$$\begin{aligned} C &= \pi d \\ \frac{C}{\pi} &= \frac{\pi d}{\pi} \\ \frac{C}{\pi} &= d \\ d &= \frac{C}{\pi} \end{aligned} \quad \text{or} \quad \begin{aligned} C &= \pi d \\ \pi d &= C \\ \frac{\pi d}{\pi} &= \frac{C}{\pi} \\ d &= \frac{C}{\pi} \end{aligned}$$

b.

$$\begin{aligned} P &= a+b+c \\ P-b &= a+b+c-b \\ P-b &= a+c \\ P-b-c &= a+c-c \\ P-b-c &= a \\ a &= P-b-c \end{aligned} \quad \text{or} \quad \begin{aligned} P &= a+b+c \\ a+b+c &= P \\ a+b+c-b-c &= P-b-c \\ a &= P-b-c \end{aligned}$$

c.

$$\begin{aligned} A &= \frac{1}{2}ab \\ 2(A) &= 2\left(\frac{1}{2}ab\right) \\ 2A &= ab \\ \frac{2A}{a} &= \frac{ab}{a} \\ \frac{2A}{a} &= b \\ b &= \frac{2A}{a} \end{aligned} \quad \text{or} \quad \begin{aligned} A &= \frac{1}{2}ab \\ \frac{1}{2}ab &= A \\ 2\left(\frac{1}{2}ab\right) &= 2(A) \\ ab &= 2A \\ \frac{ab}{a} &= \frac{2A}{a} \\ b &= \frac{2A}{a} \end{aligned}$$

d.

$$\begin{aligned} V &= \ell wh \\ \frac{V}{wh} &= \frac{\ell wh}{wh} \\ \frac{V}{wh} &= \ell \\ \ell &= \frac{V}{wh} \end{aligned} \quad \text{or} \quad \begin{aligned} V &= \ell wh \\ \ell wh &= V \\ \frac{\ell wh}{wh} &= \frac{V}{wh} \\ \ell &= \frac{V}{wh} \end{aligned}$$

e.

$$A = \frac{1}{2}(b_1 + b_2) \times h$$

$$2(A) = 2\left[\frac{1}{2}(b_1 + b_2) \times h\right]$$

$$2A = (b_1 + b_2) \times h$$

$$\frac{2A}{(b_1 + b_2)} = \frac{(b_1 + b_2) \times h}{(b_1 + b_2)}$$

$$\frac{2A}{b_1 + b_2} = h$$

$$h = \frac{2A}{b_1 + b_2}$$

14. a.

$$d = \frac{C}{\pi}$$

$$= \frac{27.004}{\pi}$$

$$\doteq 8.6 \text{ cm}$$

b.

$$a = P - b - c$$

$$= 48.5 - 7.6 - 19.4$$

$$= 21.5 \text{ cm}$$

c.

$$b = \frac{2A}{a}$$

$$= \frac{2(63.6)}{10.6}$$

$$= \frac{127.2}{10.6}$$

$$= 12 \text{ cm}$$

d.

$$\ell = \frac{V}{wh}$$

$$= \frac{336}{8.4(3.2)}$$

$$= \frac{336}{26.88}$$

$$= 12.5 \text{ cm}$$

e.

$$h = \frac{2A}{b_1 + b_2}$$

$$= \frac{2(126)}{12 + 18}$$

$$= \frac{252}{30}$$

$$= 8.4 \text{ cm}$$

15. Both formulas contain the same variables. In the first formula, you are solving for distance in terms of velocity and time. In the second formula, you are solving for velocity in terms of distance and time. They are variations of the same formula—either can be rearranged to form the other.

16. Method 1

Rearrange the formula to isolate n.

$$C = 50 + 0.1n$$

$$C - 50 = 50 + 0.1n - 50$$

$$C - 50 = 0.1n$$

$$\frac{C - 50}{0.1} = \frac{0.1n}{0.1}$$

$$\frac{C - 50}{0.1} = n$$

$$n = \frac{C - 50}{0.1}$$

Substitute the known values into the formula.

$$n = \frac{350 - 50}{0.1}$$
$$= \frac{300}{0.1}$$
$$= 3000$$

The hockey team printed 3000 tickets.

Method 2

Substitute the known values into the formula; solve for n.

$$C = 50 + 0.1n$$
$$350 = 50 + 0.1n$$
$$350 - 50 = 50 + 0.1n - 50$$
$$300 = 0.1n$$
$$\frac{300}{0.1} = \frac{0.1n}{0.1}$$
$$3000 = n$$

The hockey team printed 3000 tickets.

Now Try This

17. If the length and width of the photograph are doubled, the area will be four times the original; thus, the cost of the enlargement will be four times the cost of the original.

Looking Back

18. Answers will vary. Letters should include the reasons found in the portion of a sample letter that follows:

Hi Jadah,

I am taking math in high school and learning how to use formulas. This may seem funny coming from me, but I am getting a new insight into formulas. Formulas are important for several reasons:

- Formulas express relations in a general form. For example, $A = \ell w$ applies to all rectangles.
- Formulas express relations in a concise way. Imagine trying to express $A = \frac{1}{2}(b_1 + b_2)h$ in words.
- Formulas can be rearranged. This way, once a relation is expressed as a formula, the value of any one of the variables can be expressed in terms of the other variables. If the values for the other variables are known, the value of the "missing" variable can be determined. Take the formula for volume, $V = \ell wh$, for example; it follows that $\ell = \frac{V}{wh}$, $w = \frac{V}{\ell h}$, and $h = \frac{V}{\ell w}$.

I've worked with formulas before, but this year, for the first time, I can appreciate their value. Seeing that formulas contain so much information in such compact forms got me pretty excited—well, at least interested.

How are you doing? I haven't heard how your cycling trip between Jasper and Banff went. Any bears?

How are your studies going?

Section 2: Activity 3

1. a. Clear the denominator; then solve for m.

$$\frac{3m}{9} = \frac{7}{6}$$

$$\overset{2}{\cancel{18}}\left(\frac{3m}{\cancel{9}_{1}}\right) = \overset{3}{\cancel{18}}\left(\frac{7}{\cancel{6}_{1}}\right) \quad \leftarrow \text{The LCM of 9 and 6 is 18.}$$

$$6m = 21$$

$$\frac{6m}{6} = \frac{21}{6}$$

$$m = 3.5$$

b. Clear the denominators; then solve for y.

$$\frac{y+2}{5} = \frac{2y+1}{9}$$

$$\overset{9}{\cancel{45}}\left(\frac{y+2}{\cancel{5}_{1}}\right) = \overset{5}{\cancel{45}}\left(\frac{2y+1}{\cancel{9}_{1}}\right) \quad \leftarrow \text{The LCM of 5 and 9 is 45.}$$

$$9(y+2) = 5(2y+1)$$

$$9y + 18 = 10y + 5$$

$$9y + 18 - 18 = 10y + 5 - 18$$

$$9y = 10y - 13$$

$$9y - 10y = 10y - 13 - 10y$$

$$-y = -13$$

$$y = 13$$

c.

$$\frac{2d}{5} + \frac{d}{10} = \frac{15}{2}$$

$$10\left(\frac{2d}{5} + \frac{d}{10}\right) = 10\left(\frac{15}{2}\right) \quad \leftarrow \text{Multiply both sides by the LCM, 10.}$$

$$\overset{2}{\cancel{10}}\left(\frac{2d}{\cancel{5}_{1}}\right) + \overset{1}{\cancel{10}}\left(\frac{d}{\cancel{10}_{1}}\right) = \overset{5}{\cancel{10}}\left(\frac{15}{\cancel{2}_{1}}\right)$$

$$4d + d = 75$$

$$5d = 75$$

$$d = 15$$

d.

$$\frac{2n}{5} = \frac{n-6}{4}$$

$$\overset{4}{\cancel{20}}\left(\frac{2n}{\cancel{5}_{1}}\right) = \overset{5}{\cancel{20}}\left(\frac{n-6}{\cancel{4}_{1}}\right) \quad \leftarrow \text{Multiply both sides by the LCM, 20.}$$

$$4(2n) = 5(n-6)$$

$$8n = 5n - 30$$

$$8n - 5 = 5n - 30 - 5n$$

$$\frac{3n}{3} = \frac{-30}{3}$$

$$n = -10$$

e.

$$\frac{n}{2}+\frac{n}{3}=n-\frac{1}{8}$$

$$24\left(\frac{n}{2}+\frac{n}{3}\right)=24\left(n-\frac{1}{8}\right)$$ ← Multiply both sides by the LCM, 24.

$$\cancel{24}^{12}\left(\frac{n}{\cancel{2}_1}\right)+\cancel{24}^{8}\left(\frac{n}{\cancel{3}_1}\right)=24(n)-\cancel{24}^{3}\left(\frac{1}{\cancel{8}_1}\right)$$

$$12n+8n=24n-3$$

$$20n=24n-3$$

$$20n-24n=24n-3-24n$$

$$-4n=-3$$

$$\frac{-4n}{-4}=\frac{-3}{-4}$$

$$n=\frac{3}{4}$$

f. **Hint:** A common denominator for the equation is $2n(n+3)$. Multiply through by this expression.

$$\frac{5}{n+3}=\frac{9}{2n}$$

$$2n\cancel{(n+3)}\left(\frac{5}{\cancel{n+3}}\right)=\cancel{2n}(n+3)\left(\frac{9}{\cancel{2n}}\right)$$

$$10n=9n+27$$

$$10n-9n=9n+27-9n$$

$$n=27$$

Later in this activity, you will learn more about the method to use when solving similar equations.

g.

$$\frac{a}{2}+\frac{a}{3}+\frac{a}{4}+\frac{a}{6}=15$$

$$12\left(\frac{a}{2}+\frac{a}{3}+\frac{a}{4}+\frac{a}{6}\right)=12(15)$$

$$\cancel{12}^{6}\left(\frac{a}{\cancel{2}_1}\right)+\cancel{12}^{4}\left(\frac{a}{\cancel{3}_1}\right)+\cancel{12}^{3}\left(\frac{a}{\cancel{4}_1}\right)+\cancel{12}^{2}\left(\frac{a}{\cancel{6}_1}\right)=180$$

$$6a+4a+3a+2a=180$$

$$15a=180$$

$$\frac{15a}{15}=\frac{180}{15}$$

$$a=12$$

2. Let n be the total value of Mr. Johnson's estate. Thus, Arlene's share is $\frac{n}{2}$, Brad's share is $\frac{n}{4}$, and Jill's share is $\frac{n}{5}$.

The three charities' share totalled $3\times 3400=\$10\ 200$.

Write the equation.

$$\frac{n}{2}+\frac{n}{4}+\frac{n}{5}+10\ 200=n$$

Multiply both sides of the equation by 20, the LCM of 2, 4, and 5.

$$20\left(\frac{n}{2}+\frac{n}{4}+\frac{n}{5}+10\,200\right)=20(n)$$

$$\overset{10}{\cancel{20}}\left(\frac{n}{\cancel{2}_{1}}\right)+\overset{5}{\cancel{20}}\left(\frac{n}{\cancel{4}_{1}}\right)+\overset{4}{\cancel{20}}\left(\frac{n}{\cancel{5}_{1}}\right)+20(10\,200)=20n$$

$$10n+5n+4n+204\,000=20n$$

$$19n+204\,000=20n$$

$$19n+204\,000-19n=20n-19n$$

$$204\,000=n$$

$$n=204\,000$$

The value of the estate was \$204 000.

Arlene's share was half of the estate: \$102 000
Brad's share was one-quarter of the estate: \$51 000
Jill's share was one-fifth of the estate: \$40 800

Check

The amount left to relatives was $\$102\,000+\$51\,000+\$40\,800=\$193\,800$.

The amount remaining for three charities was
$\$204\,000-\$193\,800=\$10\,200$.

Each of the three charities received $\frac{\$10\,200}{3}=\$3400$.

This last figure agrees with the information provided in the question. The answer in the solution is reasonable.

3. Let n be the number.

Write the equation and solve.

$$\frac{2}{3}n-\frac{1}{2}=\frac{5}{6}$$

$$6\left(\frac{2}{3}n-\frac{1}{2}\right)=6\left(\frac{5}{6}\right)$$

6 is the LCM of 2, 3, and 6.

$$\overset{2}{\cancel{6}}\left(\frac{2}{\cancel{3}_{1}}n\right)-\overset{3}{\cancel{6}}\left(\frac{1}{\cancel{2}_{1}}\right)=\overset{1}{\cancel{6}}\left(\frac{5}{\cancel{6}_{1}}\right)$$

$$4n-3=5$$

$$4n-3+3=5+3$$

$$4n=8$$

$$\frac{4n}{4}=\frac{8}{4}$$

$$n=2$$

Check

Two-thirds of this number is $\frac{2}{3}\times 2=\frac{4}{3}$.

If the number is decreased by one-half, the amount becomes

$$\frac{4}{3}-\frac{1}{2}=\frac{8}{6}-\frac{3}{6}$$

$$=\frac{5}{6}$$

The solution is correct.

4. a.

$$\frac{5}{a} = 10$$

$$a\left(\frac{5}{a}\right) = a(10)$$

$$5 = 10a$$

$$\frac{5}{10} = \frac{10a}{10}$$

$$\frac{5}{10} = a$$

$$\frac{1}{2} = a$$

$$a = \frac{1}{2}$$

b.

$$\frac{7}{a} + 1 = 8$$

$$\frac{7}{a} + 1 - 1 = 8 - 1$$

$$\frac{7}{a} = 7$$

$$a\left(\frac{7}{a}\right) = a(7)$$

$$7 = 7a$$

$$\frac{7}{7} = \frac{7 \times a}{7}$$

$$1 = a$$

c.

$$\frac{3}{r} = \frac{5}{8}$$

$$8r\left(\frac{3}{r}\right) = 8r\left(\frac{5}{8}\right)$$

$$24 = 5r$$

$$\frac{24}{5} = \frac{5}{5}r$$

$$\frac{24}{5} = r$$

$$r = \frac{24}{5}$$

d.

$$\frac{7}{x} - 2 = \frac{4}{5}$$

$$\frac{7}{x} - 2 + 2 = \frac{4}{5} + 2$$

$$\frac{7}{x} = \frac{4}{5} + \frac{10}{5}$$

$$\frac{7}{x} = \frac{14}{5}$$

$$5x\left(\frac{7}{x}\right) = 5x\left(\frac{14}{5}\right)$$

$$35 = 14x$$

$$\frac{35}{14} = \frac{14x}{14}$$

$$\frac{5}{2} = x$$

$$x = \frac{5}{2}$$

5. a. Let c be the number of containers Nadia can pick in 8 h.

Method 1

$$\frac{32}{5} = \frac{c}{8}$$

$$40\left(\frac{32}{5}\right) = 40\left(\frac{c}{8}\right)$$

$$256 = 5c$$

$$5 = 256$$

$$\frac{5c}{5} = \frac{256}{5}$$

$$c = 51.2$$

Method 2

$$\frac{32}{5} = \frac{c}{8}$$

$$8\left(\frac{32}{5}\right) = 8\left(\frac{c}{8}\right)$$

$$\frac{256}{5} = c$$

$$51.2 = c$$

$$c = 51.2$$

Nadia can pick 51.2 (51 full) containers in 8 h.

Check

Reread the question.

Should Nadia be able to pick more or less than 32 containers in 8 h? Yes, she should be able to pick more than 32 containers in 8 h since she would be working longer.

Therefore, it is reasonable that Nadia can pick 51.2 (51 full) containers in 8 h.

b. Let h be the number of hours needed to pick 112 containers of blueberries.

Method 1

$$\frac{32}{5} = \frac{112}{h}$$

$$5h\left(\frac{32}{5}\right) = 5h\left(\frac{112}{h}\right)$$

$$32h = 560$$

$$\frac{32h}{32} = \frac{560}{32}$$

$$h = 17.5$$

Method 2

$$\frac{32}{5} = \frac{112}{h}$$

$$h\left(\frac{32}{5}\right) = h\left(\frac{112}{h}\right)$$

$$\frac{32h}{5} = 112$$

$$\frac{5}{32}\left(\frac{32h}{5}\right) = \frac{5}{32}\left(112\right)$$

$$h = \frac{35}{2}$$

$$h = 17.5$$

Nadia will take 17.5 h to pick 112 containers.

Check

17.5 h is more than 3 times 5 h. Is 112 containers more than 3 times 32 containers? Yes.

Therefore, it will take Nadia 17.5 h to pick 112 containers of wild blueberries is a reasonable answer to the question.

6. Let n be the smaller part. Thus, $n+4$ is the larger part.

Method 1

$$\frac{n}{n+4} = \frac{5}{7}$$

$$7(n+4)\left(\frac{n}{n+4}\right) = 7(n+4)\left(\frac{5}{7}\right)$$

$$7n = 5n + 20$$

$$7n - 5n = 5n + 20 - 5n$$

$$2n = 20$$

$$\frac{2n}{2} = \frac{20}{2}$$

$$n = 10$$

Method 2

$$\frac{n}{n+4} = \frac{5}{7}$$

$$(n+4) \times \frac{n}{(n+4)} = \frac{5(n+4)}{7}$$

$$n = \frac{5n+20}{7}$$

$$n \times 7 = \frac{(5n+20)}{7} \times 7$$

$$7n = 5n + 20$$

$$7n - 5n = 5n + 20 - 5n$$

$$2n = 20$$

$$\frac{2n}{2} = \frac{20}{2}$$

$$n = 10$$

Now, find the larger part.

$$n + 4 = 10 + 4$$

$$= 14$$

Check

Is the sum of 10 and 14 equal to 24? Yes.

Do the numbers 10 and 14 have a ratio of 5 to 7? Yes.

Therefore, the smaller part is 10 and the larger part is 14.

Looking Back

7. $\frac{x}{3}-\frac{x}{6}=2-\frac{x}{9}$

Multiply each side by 18, the lowest common multiple of the three denominators.

$$\cancel{18}^{6}\left(\frac{x}{\cancel{3}_{1}}\right)-\cancel{18}^{3}\left(\frac{x}{\cancel{6}_{1}}\right)=18(2)-\cancel{18}^{2}\left(\frac{x}{\cancel{9}_{1}}\right)$$

$$6x-3x=36-2x$$

$$3x=36-2x$$

$$3x+2x=36-2x+2x$$

$$5x=36$$

$$\frac{5x}{5}=\frac{36}{5}$$

$$x=7.2$$

Remember: Every element on both sides of the equation must be multiplied by 18.

Credits

All clip art drawings are commercially owned.

Welcome Page
PhotoDisc, Inc.

Page
1 PhotoDisc, Inc.
3 Corel Corporation
4 **left:** PhotoDisc, Inc.
right: PhotoDisc, Inc.
5 **left:** PhotoDisc, Inc.
6 **upper left:** PhotoDisc, Inc.
lower left: PhotoDisc, Inc.
7 Corbis
10 PhotoDisc, Inc.
14 **right:** PhotoDisc, Inc.
21 PhotoDisc, Inc.
22 **left:** PhotoDisc, Inc.
right: PhotoDisc, Inc.
24 **right:** EyeWire, Inc.
27 **upper left:** PhotoDisc, Inc.
lower left: PhotoDisc, Inc.
right: PhotoDisc, Inc.
31 PhotoDisc, Inc.
33 Corel Corporation
34 PhotoDisc, Inc.
35 PhotoDisc, Inc.
41 **left:** PhotoDisc, Inc.
42 EyeWire, Inc.
43 Nova Development Corporation
44 **upper left:** PhotoDisc, Inc.
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45 **right:** PhotoDisc, Inc.
47 EyeWire, Inc.
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58 PhotoDisc, Inc.
61 **upper left:** PhotoDisc, Inc.
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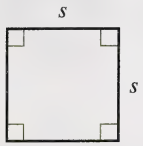
69 PhotoDisc, Inc.
70 PhotoDisc, Inc.
74 EyeWire, Inc.
77 PhotoDisc, Inc.
86 PhotoDisc, Inc.
87 **left:** PhotoDisc, Inc.
88 PhotoDisc, Inc.

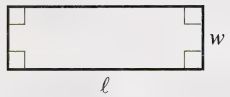
Formula Sheet

Area

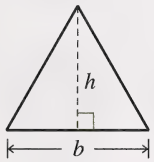
$A = s^2$ area of a square



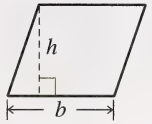
$A = \ell w$ area of a rectangle



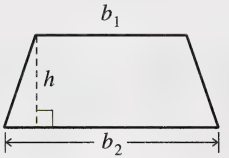
$A = \frac{1}{2}bh$ area of a triangle



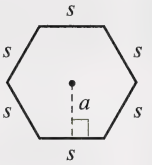
$A = bh$ area of a parallelogram



$A = \frac{(b_1 + b_2)h}{2}$ area of a trapezoid



$A = \frac{nsa}{2}$ area of a regular polygon



n = number of sides,
s = length of each side, and
a = length of the apothem

$A = \pi r^2$ area of a circle

Volume

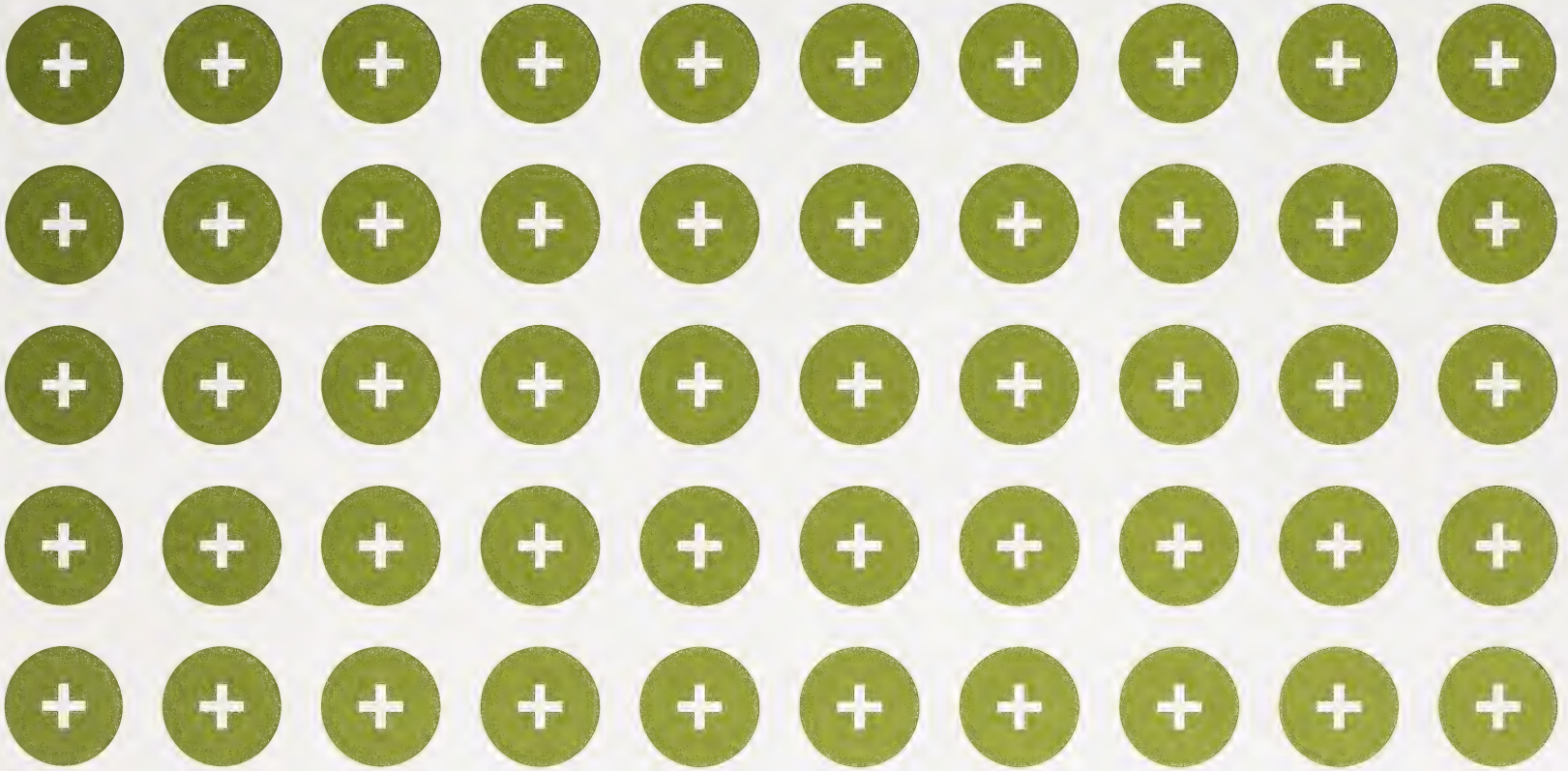
$V = BH$ volume of a prism

$V = BH$ volume of a cylinder

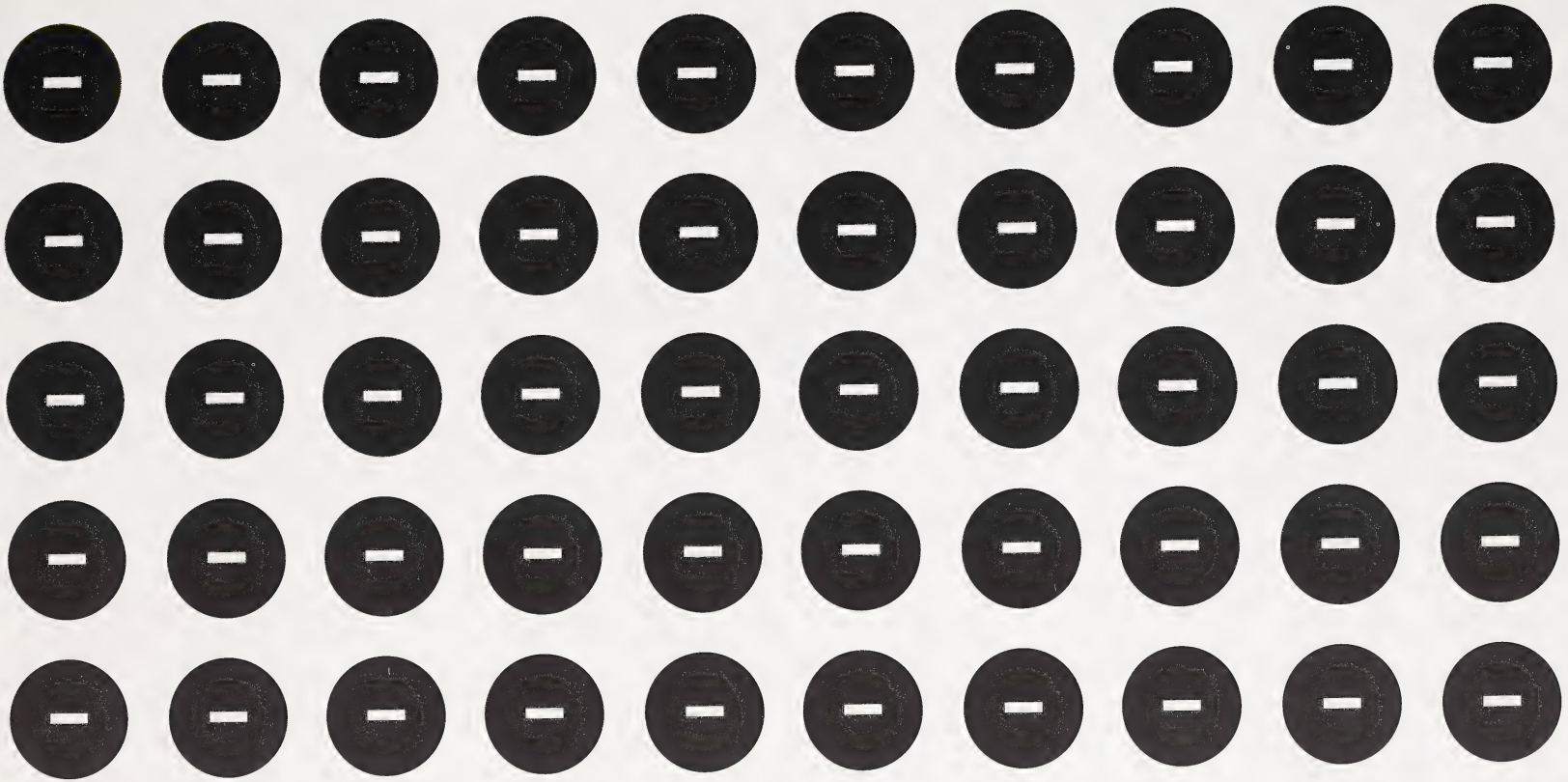
$V = \frac{1}{3}BH$ volume of a cone

$V = \frac{1}{3}BH$ volume of a pyramid

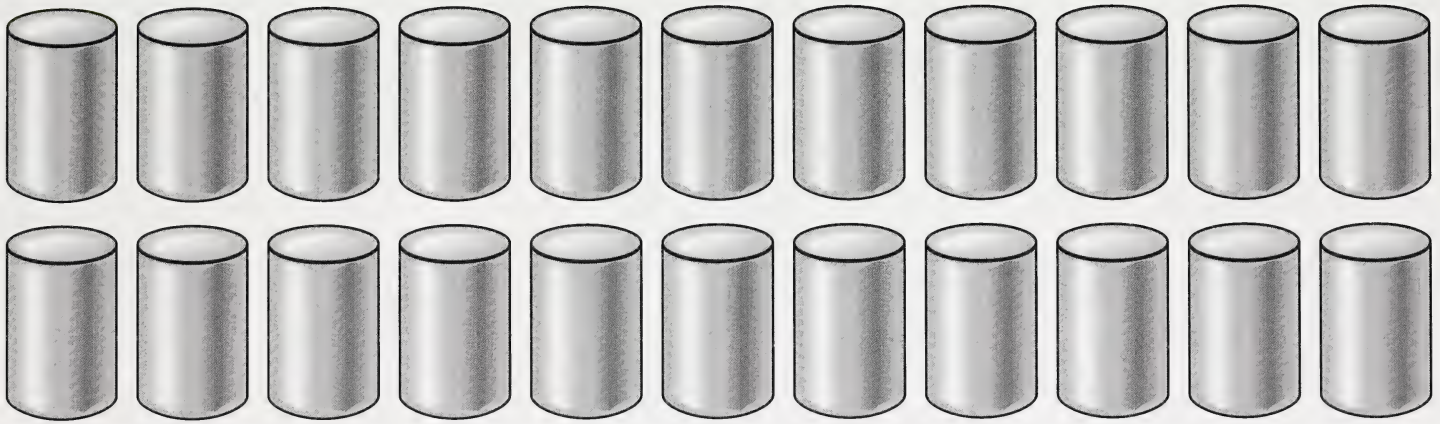
Positive Counters



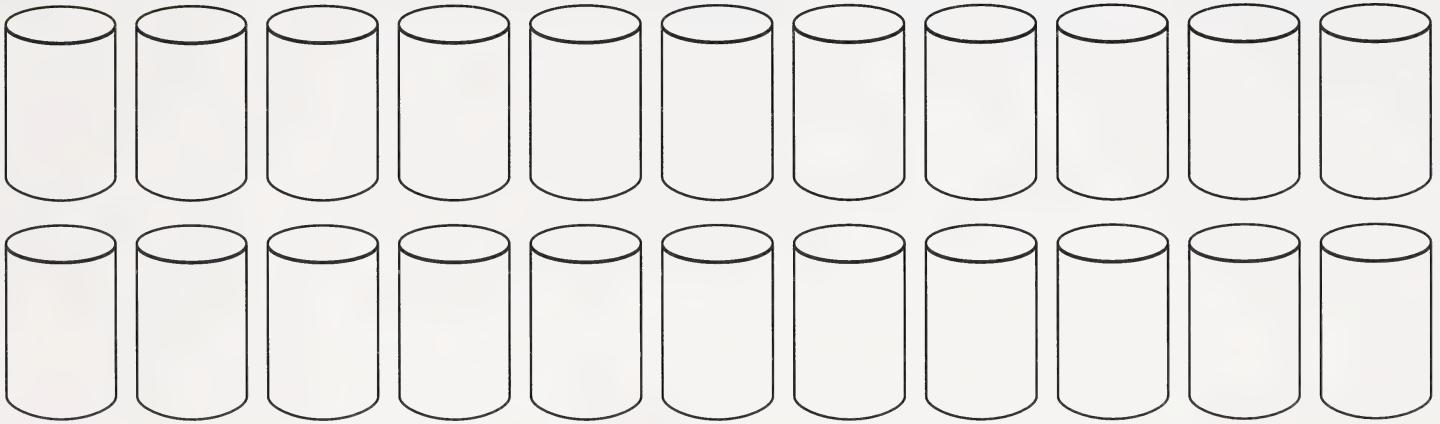
Negative Counters



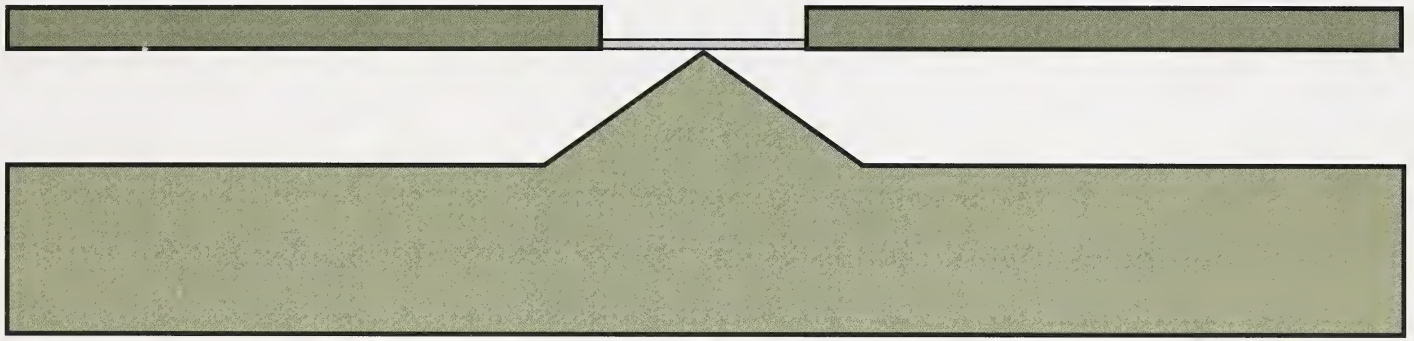
Positive Cylinders



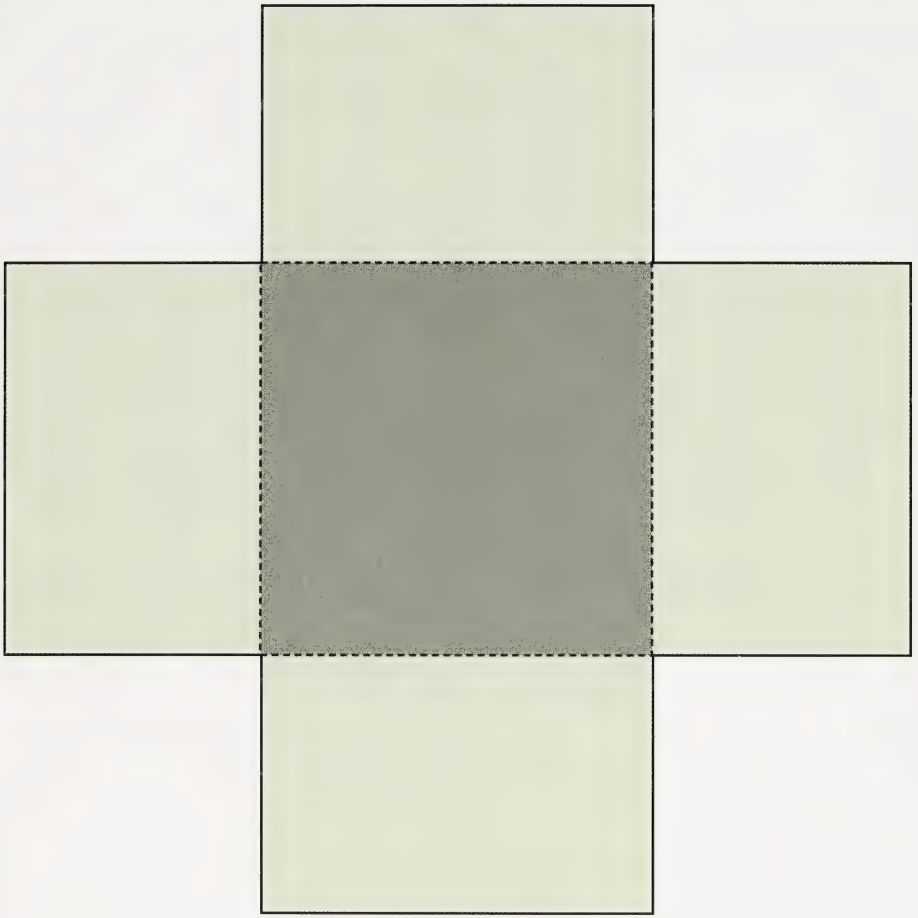
Negative Cylinders



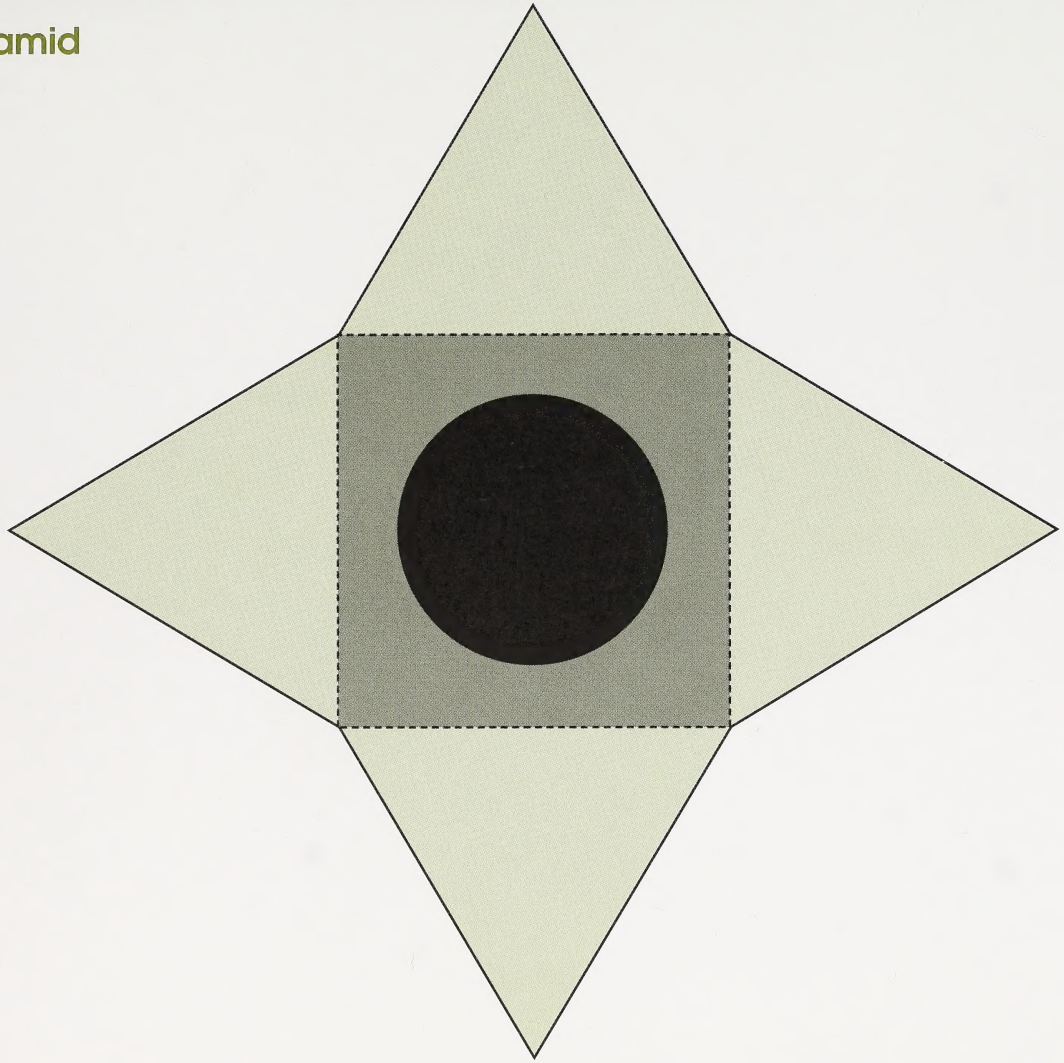
Equation Scale

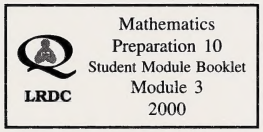


Rectangular Prism



Rectangular Pyramid





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